A STATISTICAL FRAMEWORK FOR IMAGE-BASED RELIGHTING

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ABSTRACT

With image-based relighting (IBL), one can render realistic relit images of a scene without prior knowledge of object geometry in the scene. However, traditional IBL methods require a large number of basis images, each corresponding to a lighting pattern, to estimate the surface reflectance function (SRF) of the scene. In this paper, we present a statistical approach to estimating the SRF which requires fewer basis images. We formulate the SRF estimation problem in a signal reconstruction framework. We use the principal component analysis (PCA, [1]) to show that the most effective lighting patterns for the data acquisition process are the eigenvectors of the covariance matrix of the SRFs, corresponding to the largest eigenvalues. In addition, we show that for typical SRFs, especially when the objects have Lambertian surfaces, DCT-based lighting patterns perform as well as the optimal PCA-based lighting patterns. We compare SRF estimation performance of the statistical approach with traditional IBL techniques. Experimental results show that the statistical approach can achieve better performance with fewer basis images.

1. INTRODUCTION

Image-based relighting (IBL) represents a class of techniques that synthesize images of the scene under novel lighting conditions, given a set of basis images. IBL has a great advantage that prior knowledge of object geometry in the scene is not needed for relighting. IBL is applied many applications to such as realistic visualization of the real objects/scenes in a virtual environment and movie special effects.

Prior work for IBL has been done by reconstructing a 2D light mapping function for each pixel of an image, such as the *plenoptic illumination function* [2], *a reflected irradiance field* [3], *an environment matte* [4, 5], or *a reflectance field* [6]. These techniques are different in representation but identical in concept. In this paper, we define a function for each pixel and call it the Surface Reflectance Function (SRF); the SRF is a weighting function that represents the contributions from point light sources to individual pixel on the image plane (Figure 1).

Nimeroff et al. [7] explained the concept of the illumination as the linear combination of weighted basis images lit by steerable functions. Environment matting, extension to relighting applications, attempted to design lighting patterns to obtain the most efficient basis images for relighting. Zongker et al. [4] introduced an approach to estimating an environment matte of specular and transparent objects using basis images taken with Gray-code lighting patterns. Chuang et al. [8] extended this [4] technique for higher accuracy and real-time capturing. Chuang et al. [8] use differently

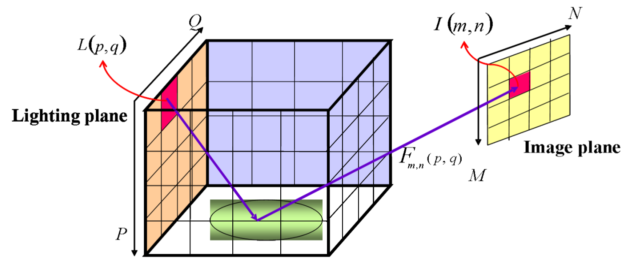


Fig. 1. The SRF model

oriented Gaussian stripe patterns instead of Gray-code patterns and estimate matte parameters assuming that an environment matte is a Gaussian function. Chuang et al. [8] also provided a method to extract an environment matte from single basis image under certain scene constraints. Peers et al. [5] improved the efficiency of acquiring an environment matte of a scene by using wavelet lighting patterns. They measured the importance value of an applied pattern by taking the norm of the corresponding image. By learning from previously recorded images, they could make a decision of the most important lighting patterns among wavelets. Our work differs from Peer et al. [5] in the sense that we design the lighting patterns based on data driven statistics of SRFs. Peer et al. [5] showed that smartly chosen lighting patterns, wavelet basis functions, improved the accuracy of estimating an environment matte. However, our proposed algorithm to estimate a SRF uses much fewer basis images than [4, 8, 5].

The main contribution of our work is to show that data driven statistics can significantly improve the efficiency of relighting with highly satisfying visual quality. All procedure of proposed method is described in Figure 2.

Our proposed algorithm has two stages; a training stage and a testing stage. First, for the training, we collect true SRF statistics from many synthetic images categorized by surface properties (Lambertian, specular) by ray tracer (POV-RAY) and perform PCA [1] on the SRFs (Figure 2.(1)). PCA results show that SRFs are a highly correlated data set so that only a few eigenvectors dominate the energy distribution of the SRF. Based on this observation, we propose a method to design lighting patterns, to be the eigenvectors of the covariance matrix of the SRFs.

Second, for the testing, designed lighting patterns from the training stage (Figure 2.(1)) are applied to acquire basis images. Basis images are used to synthesize relit images with novel light-

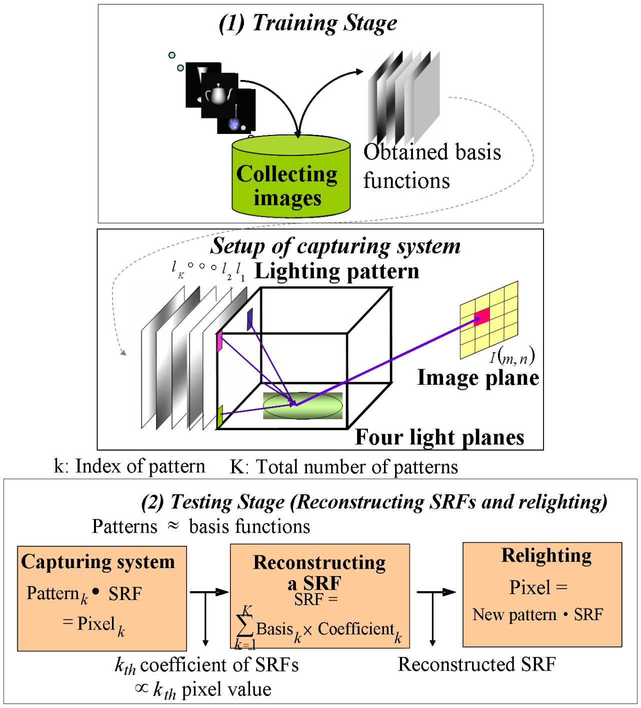


Fig. 2. Flow of proposed algorithm

ing patterns (Figure 2.(2)).

In this paper, we show the surface properties effects on the data distribution of SRFs based on the PCA results. It is important to note that the SRFs can be modelled as an AR(1) process with a high correlation parameter so that the PCA basis functions can be replaced by DCT basis functions [9]. Especially SRFs from Lambertian surface objects fit better into an AR(1) model than SRFs from Specular surface objects. We can see that DCT-based lighting patterns provide better performance for Lambertian surface objects than Specular surface objects in the performance comparison (Figure 5).

In the following section, we explain our algorithm for performing the reconstruction of SRFs from basis images and relighting using reconstructed SRFs (Section 2, Figure 2.(2)). Section 3 contains practical implementation issues. As a conclusion, we compare the performance of our reconstructed SRFs with other literature [4, 8, 5] and present the relit images using reconstructed SRFs and images of same object with real lighting together (Section 4).

1.1. Related Work

There are many related IBL work that studied a sampling and compression [2, 3, 10] of SRF. Lin et al. [3] introduced the reflected irradiance field and solved the minimum sampling problem of the reflected irradiance field. Wong [2] extracted the lighting factor from the plenoptic function and named it the plenoptic illumination function. Wong [2] discussed about the compression issue of plenoptic illumination function. Ho et al. [10] proposed the compression algorithm for IBL, after acquiring complete SRFs data set, based on PCA [1].

2. ALGORITHM DESCRIPTION

In this section, we define the illumination of a pixel on the image plane with the rendering equation and develop a mathematical framework to solve a SRF. A SRF is defined as a weighting function from light sources to the radiance value. Therefore, the radiance value reflected by the surface is the inner product value of a SRF and a lighting pattern [4, 8, 5, 7, 3, 6, 2].

$$I(m,n) = \sum_{p=1}^{\mathcal{P}}\sum_{q=1}^{\mathcal{Q}} \mathcal{F}_{m,n}(p,q)L(p,q) \quad (1)$$

p, q: The index of the light plane, $1 \le p \le \mathcal{P}, 1 \le q \le \mathcal{Q}$
m, n: The index of the image plane, $1 \le n \le \mathcal{N}, 1 \le m \le \mathcal{M}$
$I(m,n)$: Radiance value at (m,n) pixel where $\mathcal{L}_{in}$ is a corresponding lighting pattern
$\mathcal{F}_{m,n}$: The SRF for a pixel (m,n),$0 \le \mathcal{F}_{m,n} \le 1$
$L(p,q)$: Incoming radiance at (p,q) from the light plane

We can cascade elements of a SRF function and a lighting pattern into vectors so that the rendering equation turns into Equation 2.

$$I(m,n) = \mathbf{F}^T\mathbf{L} \quad (2)$$

$\mathbf{F}$ is a $\mathcal{PQ}\times 1$ vector and $\mathbf{L}_{in}$ is a $\mathcal{PQ}\times 1$ vector. Our goal is to find $\mathbf{F}$, a SRF. Assuming that $\mathbf{F}$ is a random vector and a statistics of $\mathbf{F}$ are given, we can obtain most representative basis functions of $\mathbf{F}$ using PCA [1]. Therefore, $\mathbf{F}$ can be decomposed by the linear combination of basis functions multiplied by a set of coefficients (Equation 3).

$$\mathbf{F} = c_1\mathbf{e}_1 + c_2\mathbf{e}_2 + ... + c_{PQ}\mathbf{e}_{PQ} + \mathbf{m} \quad (3)$$

$\mathbf{e}_k$: *k_{th} basis function*
c_k: *k_{th} coefficient corresponding to k_{th} basis function*
$\mathbf{m}$: *A mean vector from statistics*

In this paper, we set $\mathcal{P} = \mathcal{Q} = 64$. From the PCA of the training data set (Figure 2. (1)), we observe that only a few basis functions dominate most of the energy distribution of the SRFs (Figure 3).

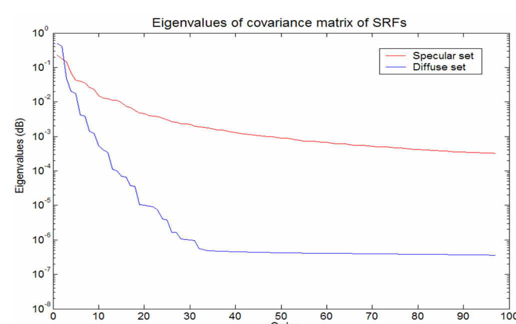
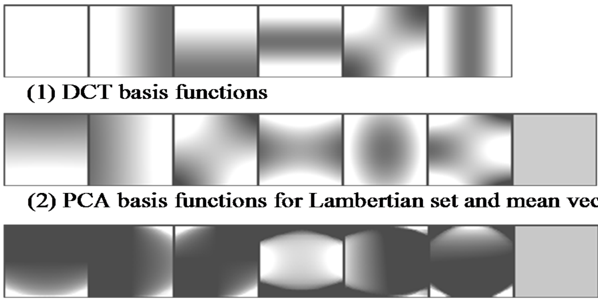


Fig. 3. Eigenvalues of the covariance matrix of SRFs

From the Lambertian set of surface objects, we find that 99.14% of energy is preserved within the first six eigenvectors out of all eigenvectors. For the specular set, 70% of energy stays within the first six eigenvectors and 90% within the first forty three eigenvectors. Therefore, $\mathbf{F}$ can be approximated by

$$\tilde{\mathbf{F}} \approx c_1\mathbf{e}_1 + c_2\mathbf{e}_2 + ... + c_K\mathbf{e}_K + \mathbf{m}, \mathcal{K} \ll \mathcal{PQ} \quad (4)$$



(1) DCT basis functions

(2) PCA basis functions for Lambertian set and mean vector

(3) PCA basis functions for specular set and mean vector

Fig. 4. First six designed lighting patterns

Therefore, our goal becomes the signal reconstruction using a minimum number of basis functions. To solve $\mathbf{F}$, we substitute $\mathbf{L}_{in}$ in Equation 2 with $\mathbf{e}_k$.

$$\begin{aligned} I_k(m,n) &= (c_1\mathbf{e}_1 + ... + c_k\mathbf{e}_k + ... + c_K\mathbf{e}_K + \mathbf{m})^T\mathbf{e}_k \\ &= c_k + \mathbf{m}^T\mathbf{e}_k \end{aligned} \tag{5}$$

Note that basis functions are orthogonal to each other and they have a unit norm. From Equation 5, a coefficient, c_k, is simply calculated from a radiance value at pixel (m,n) if a corresponding basis function $\mathbf{e}_k$ is applied as a lighting pattern. In order to reconstruct $\mathbf{F}$ without loss, we will need PQ number of basis functions. However, PQ is typically a numerous number, which is 4096 in this paper. We will select a few basis functions, corresponding to the largest eigenvalues, as lighting patterns to obtain the basis images and show the relighting results of the synthetic scenes in section 4. Six eigenvectors corresponding to the six largest eigenvalues and a mean vector from each set of statistics are presented in Figure 4 (1), (2), (3). If we collect enough data from many different objects, we obtain a constant mean vector from PCA [1] as shown in Figure 4 (2), 4 (3). The first six DCT basis functions are shown in Figure 4 (1). From Figure 4, we can observe that the basis functions from the Lambertian set are patterns with less contrast than the basis functions from the specular set of objects. This is as expected because the SRFs from the Lambertian set have a wider and flatter characteristics than the SRFs from the specular set.

Equation 5 shows the way to calculate coefficients corresponding to the basis functions. We apply calculated coefficients from Equation 5 to Equation 4 to obtain the reconstructed SRFs. Relighting a pixel is done by taking inner product of SRF with a novel lighting pattern as shown in Equation 2.

3. IMPLEMENTATION DETAILS

3.1. Displaying Lighting Patterns

In section 2, we design the optimal lighting patterns as basis functions of the covariance matrix of SRFs. To apply them as lighting patterns, we have to fit them into the range of an image. Since derived lighting patterns contain negative values and have a unit norm, it is necessary to scale and shift a basis function by following operation.

$$\mathbf{L}_k = \frac{255}{S_1}(\mathbf{e}_k + |min(\mathbf{E})|\mathbf{1}) \tag{6}$$

$\mathbf{k}$ is the index of the basis, $\mathbf{e}_k$ is a $\mathbf{k}_{th}$ basis function, $\mathbf{E}$ is a basis matrix $[\mathbf{e}_1\mathbf{e}_2...\mathbf{e}_K]$, $\mathbf{L}_{in,k}$ is the $\mathbf{k}_{th}$ lighting pattern, $\mathbf{1}$ is a one vector and S_1 is a constant value as $|max(\mathbf{E})| + |min(\mathbf{E})|$. Equation 6 describes the way to scale and shift a basis in order to make a pattern.

3.2. The Reconstruction of SRFs

In this section, we present the way to calculate the coefficients, c_k, to reconstruct the SRF.

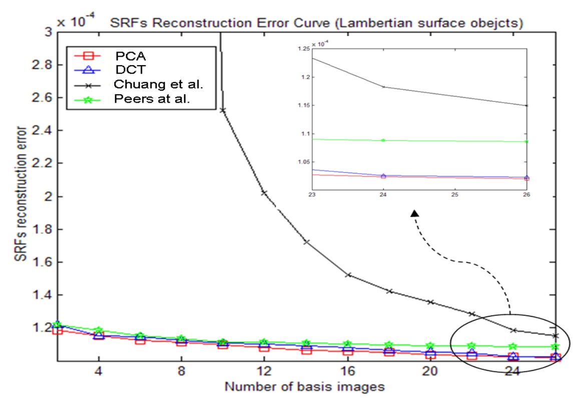
$$c_k = \frac{S_1}{255}I_k(m,n) - \mathbf{e}_k{}^T\mathbf{m} - |min(\mathbf{E})|\mathbf{F}^T\mathbf{1} \tag{7}$$

$\mathbf{F}^T\mathbf{1}$ in Equation 7 is $\frac{I_{gray}(m,n)}{128}$, where $I_{gray}(m,n)$ is a pixel value captured with the solid gray lighting pattern.

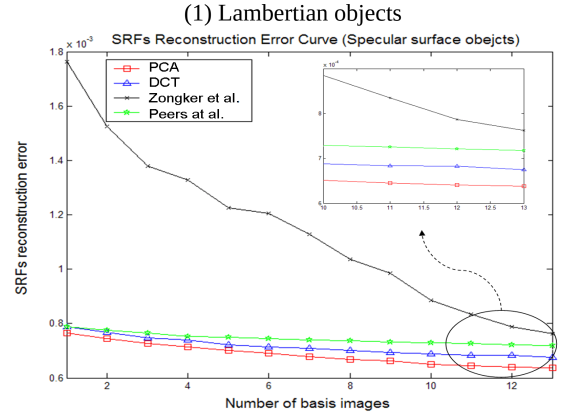
4. EXPERIMENT RESULTS

4.1. The Performance of SRFs Reconstruction

The proposed algorithm is compared with other relighting algorithms, which use an environment matte [4, 8, 5]. We apply a DCT-based approach and a PCA-based approach, and evaluate the performance. Note that training objects to create PCA basis functions and testing objects are different except the surface properties. In Figure 5 (1), we generate the SRFs reconstruction error curve using ten different diffused objects by increasing the number of lighting patterns for our algorithm and method [5, 8]. Figure 5 (2) shows the performance between our proposed methods and [5, 4] when ten different specular objects are used as objects.



(1) Lambertian objects



(2) Specular objects

Fig. 5. Comparison with another relighting algorithm

From Figure 5, our proposed methods outperform others [4, 8, 5]. The range of SRFs is between zero and one. Thus, although numerically the mean square error looks small, it considerably affects on the quality of relighting. From the result shown in Figure 5, a DCT-based approach fits better for Lambertian scene objects. From the signal processing literature [9], if the signal is an AR(1) process and a model parameter, ρ, of an AR(1) model is close to 1, the optimal basis functions for the signal are DCT basis functions. From this fact, we fit SRFs into an AR(1) and verify whether a model parameter, ρ, is close to 1 or not. We check the significant level of the modified Li-McLeod portmanteau (LMP) statistic [11]. As a results, ρ estimated from Lambertain scene objects is 0.91 ± 0.11 with 95% confidence intervals and ρ estimated from Specular scene objects is 0.83 ± 0.24 with 95% confidence intervals. It shows that SRFs from Lambertain objects fit better for an AR(1) process and, therefore, a DCT-based approach fits better for Lambertain objects. Training data set to estimate an AR(1) model parameter are equivalent to training data set in Figure 2 (1).

4.2. Relit images

Relit images are presented in Figures 6 and 7. If the scene is the Lambertian, our algorithm can provide very good quality of relit images using only six basis images. For the specular scene, it is necessary to use more basis images corresponding to high order basis functions than for the Lambertian objects since SRFs for specular objects are sharper and narrower in space than SRFs of Lambertian objects. If the difference map in Figures 6 and 7 is close to the gray color, a corresponding image is close to the ground truth. Figure 6 shows that our method can generate better quality of relit images than [5], even though we apply fewer patterns, six, in comparison to [5] where 15 lighting patterns are applied. In Figure 7, we fix the number of patterns to be 15 for both algorithms and compare the quality of the relit images. We can see that relit images from a PCA-based algorithm are significantly closer to the ground truth than [5]. From relit images, we can also see that our algorithm extends relighting to larger images, 480×360 than training images, 64×64.

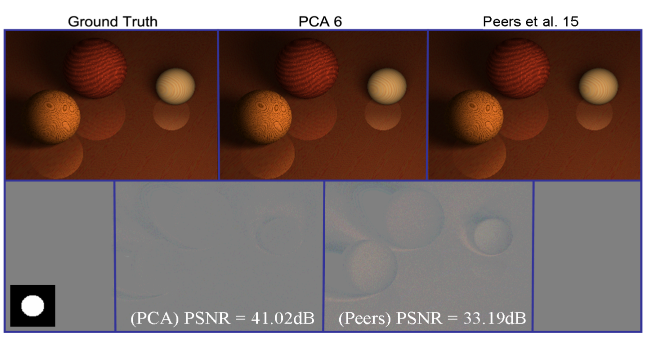


Fig. 6. (1)An object under real lighting(ground truth), (2)A relit image by our proposed method using six images, (3)A relight image by [6] using fifteen images, (4)The difference map between (1) and (2), (5)The difference map between (1) and (3)

5. CONCLUSION

In this paper, we introduce a statistical approach to reconstructing SRFs. Statistical analysis of SRFs provides optimal lighting patterns as simply the eigenvectors of the covariance matrix of the

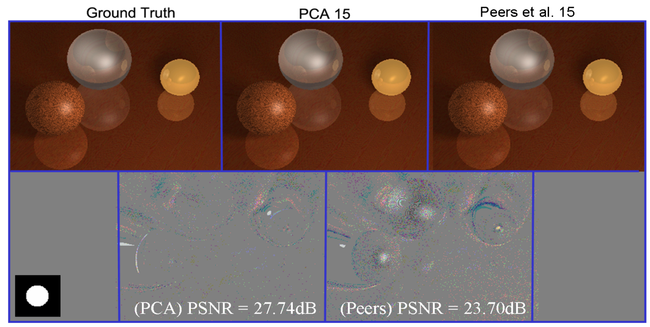


Fig. 7. (1)An object under real lighting(ground truth), (2)A relit image by our proposed method using fifteen images, (3)A relit image by [6] using fifteen images, (4)The difference map between (1) and (2), (5)The difference map between (1) and (3)

SRFs. Using optimal patterns, we are able to reconstruct SRFs using much fewer patterns with better performance than conventional relighting approaches.

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