Joint Optimization of Manifold Learning and Sparse Representations for Face and Gesture Analysis

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RIT Acknowledgements

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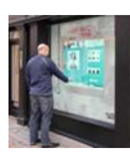
Motivation

- Facial understanding and gesture recognition are powerful enablers in intelligent vision systems.
- Potential applications include surveillance, security, entertainment, smart spaces, and human computer interfaces (HCI).
- Tomorrow's devices will need to embrace human subtleties while interacting with them in their natural conditions.

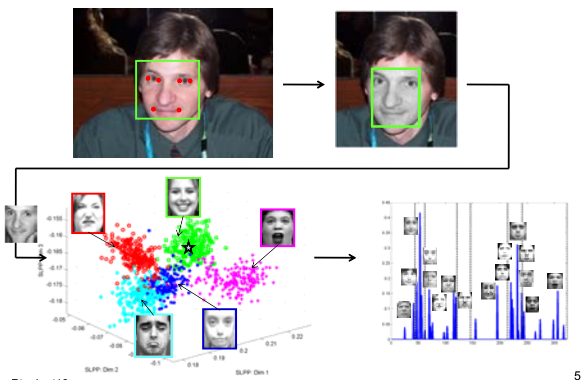
Interactive Digital Signage







Static Processing



A Few Milestones

- Yang [PAMI '07] used dimensionality reduction with SRs for classification purposes.
- Wright [PAMI '09] used SRs for best in class facial recognition.
- Zafeiriou [CVPR '10] used PCA and SR methods based on Wright for facial expression, but reported significant coefficient contamination.
- Ptucha [ICCV '11] used supervised manifold learning to minimize coefficient contamination.
- Jiang [CVPR '11,'12] used K-SVD to jointly optimize classification accuracy and more efficient dictionaries.

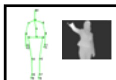
Agenda

- Introduction to Dimensionality Reduction
- Introduction to Sparse Representations
- Merging the two concepts into Manifold based Sparse Representations
- Optimizing the two concepts with LGE-KSVD
- Sample Results

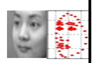
Hypothesis

- Methods based on manifold learning and sparse representations can achieve accurate, robust, and efficient classifiers for scene understanding.

Feature Extraction → **Feature Normalization** → **Manifold Learning** → **Sparse Representation** → **Classification Model** → **Temporal Filtering**



Dimensionality Reduction



- For the purpose of facial understanding, the dimensionality of a 26x20 ($\in \mathbf{R}^{520}$) pixel face image or a 82x2 ($\in \mathbf{R}^{164}$) set of ASM coordinates are artificially high.
- The high dimensionality space makes the facial understanding algorithms more complex than necessary.
- The set of 520 pixels (or 164 coordinates) actually are samples from a lower dimensional manifold that is embedded in a higher dimensional space.
- We would like to discover this lower dimensional manifold representation (to simplify our facial modeling)- a technique formally called manifold learning. [Cayton '05, Ghodsi '06]
- Given a set of inputs $x_1..x_n \in \mathbf{R}^D$, find a mapping $y_i = f(x_i)$, $y_1..y_n \in \mathbf{R}^d$; where $d < D$.

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Locality Preserving Projections* (LPP)

[He '03]

- Given a set of input points $x_1..x_n \in \mathbf{R}^D$, find a mapping $y_i = A^T x_i$, where the resulting $y_1..y_n \in \mathbf{R}^d$; where $\cancel{d < D.}$ $d << D$.
  - Same algebra as PCA, if we kept the top d eigenvectors!
- Create a fully connected adjacency graph W. Assign high weights to close/similar nodes, and low weights to far/dissimilar nodes.
  - Mimic local neighborhood structure from input to projected space.
- LPP is a linear approximation to the nonlinear Laplacian Eigenmap and is solved via the generalized eigenvector problem:

  $X L X^T a = \lambda X D X^T a$
- Where:
  - D is a diagonal matrix whose values are the column sums of W,
  - L is the Laplacian matrix: $L = D\text{-}W$,
  - a is the resulting projection matrix (== "eigenvectors") , and
  - λ is the resulting vector importance (== "eigenvalues") .

* http://www.cad.zju.edu.cn/home/dengcai/Data/DimensionReduction.html

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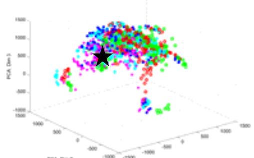
PCA vs. Supervised LPP

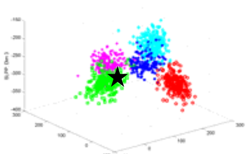
1072 samples, 26x20 pixel faces ($\mathbf{R}^{520} \rightarrow \mathbf{R}^3$)

[Lucey '10]

Top 3 dimms of PCA space.



Top 3 dimms of SLPP space.



- PCA is good at dimensionality reduction, but assumes linearity.

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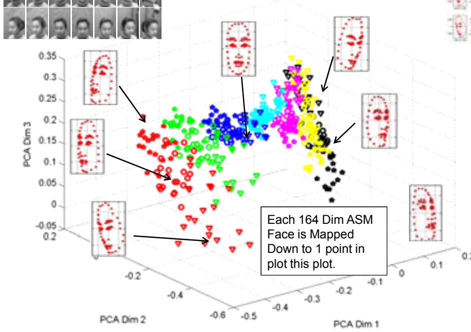
Apply Dimensionality Reduction to Pose Training Set

21 subjects, each at 21 poses



Yaw:
R=-45
G=-30
B=-15
C=0
M=15
Y=30
K=45

Pitch:
▽ = down
○ = center
* = up

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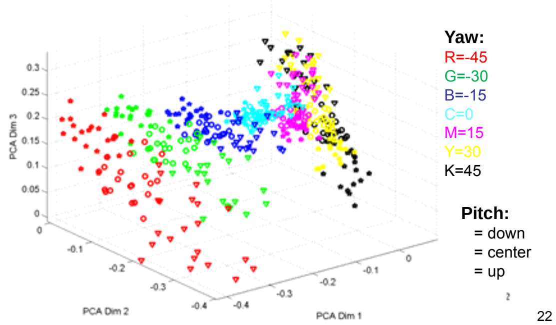
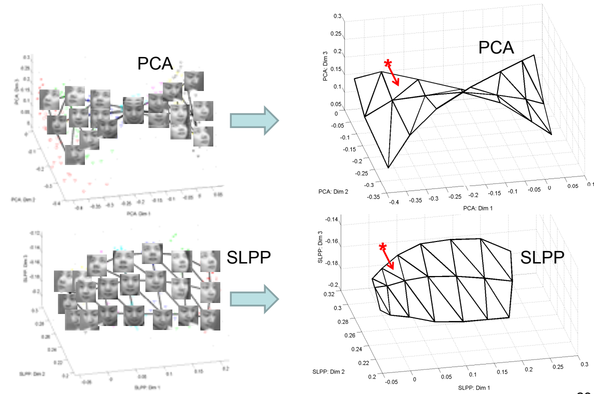
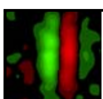
Apply Dimensionality Reduction to Pose Training Set

21 subjects, each at 21 poses



Model Manifold Surface





Actual simple cell response

Sparse Representations

- Inspired by studies of neurons in the visual cortex, the notion of Sparse Representations (SRs) has been proven applicable to a variety of scientific fields.
- For many input signals, such as natural images, only a small number of exemplars are needed to represent new test samples.
- SR gives state-of-the-art results for pattern recognition, noise reduction, super-resolution, tracking, …
- At the The First Facial Expression Recognition and Analysis Challenge (FERA2011) at FG'11:
  - 13/15 entrants used SVM, but 0/15 entrants used SR

[Images from DeAngelis, Ohzawa & Freeman, 1995]

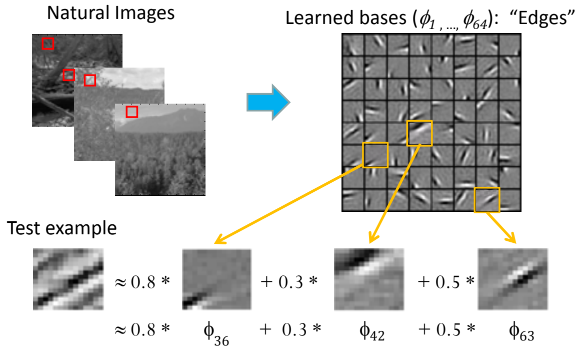
Sparse Representations

- Input: Images $x_1, x_2, \ldots, x_n$ (each $\in \mathbf{R}^{h \times w}$)
- Learn: Dictionary of bases $\phi_1, \phi_2, \ldots, \phi_k$ (each also $\in \mathbf{R}^{h \times w}$), so that each input x (and newly introduced test samples y) can be approximately decomposed as:

$$\hat{x} \approx \sum_{j=1}^{k} a_j \phi_j$$

s.t. a_j's are mostly zero ("sparse")

Sparse coding illustration [Ng ECCV '10]



[0, 0, …, 0, **0.8**, 0, …, 0, **0.3**, 0, …, 0, **0.5**, 0]
= [a_1, …, a_{64}] (feature representation)

Sparse Representations

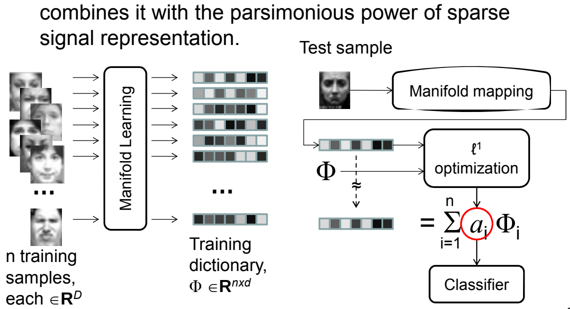
- Given y and Φ, the objective of SRs is to identify the smallest number of nonzero coefficients $a \in \mathbf{R}^n$ such that:
  $y \approx \hat{y} = \Phi\, a$.
- The solution is equivalent to the Lasso regression:

$$\hat{a} = \min\left\{ \left\|y - \Phi a\right\|^2 + \lambda \left\|a\right\|_1 \right\}$$

  where $\|a\|_1 = \Sigma\, |a|$.
- Although not differentiable like a ridge regression, the ℓ^1 minimization problem can be efficiently solved using convex optimization algorithms. [Donoho '06, Candes '06]
- Some of the fastest approaches include several variants of Least Angle Regression with lasSo (LARS). [Efron '04]

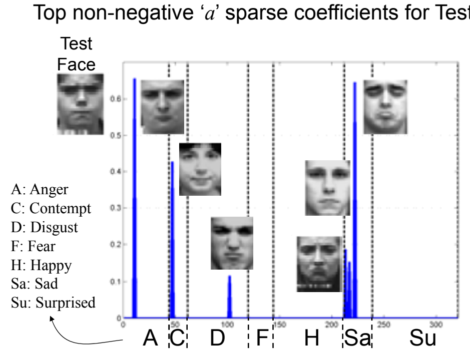
Putting it Together

- Manifold based Sparse Representations (MSR) exploit the discriminative behavior of manifold learning, and combines it with the parsimonious power of sparse signal representation.



Sparse Coefficients

Top non-negative 'a' sparse coefficients for Test "sad" Face.



Interesting…but ..how do we turn this into a classifier?

- Max peak?
- Max non-zero coefficients?
- Max Energy?

Reconstruction Error

- A reconstruction error classifier generally outperforms other methods. [Yang '07, Wright '09]
- Estimate the class, c^* of a query sample y by comparing the reconstruction error inquired when only the reconstruction coefficients a_c corresponding to a specific class c are selected.

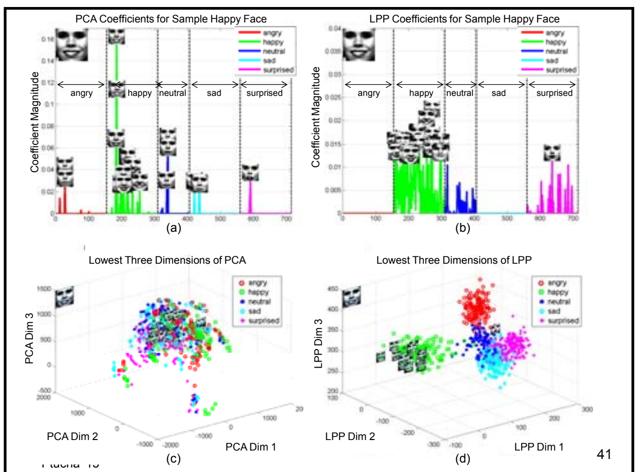
$$c^* = \arg\min_{c=1\ldots z} \|y - \Phi\, \underbrace{a_c}\|_2$$

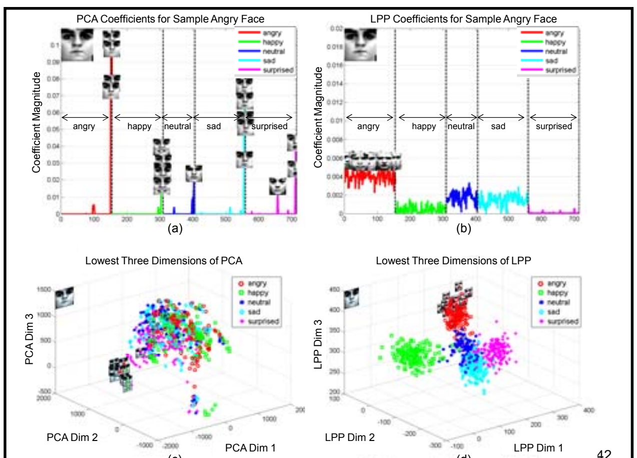
Use non-zero coefficients from all classes to estimate, $y \approx \Phi\, a$

Use non-zero coefficients from each class

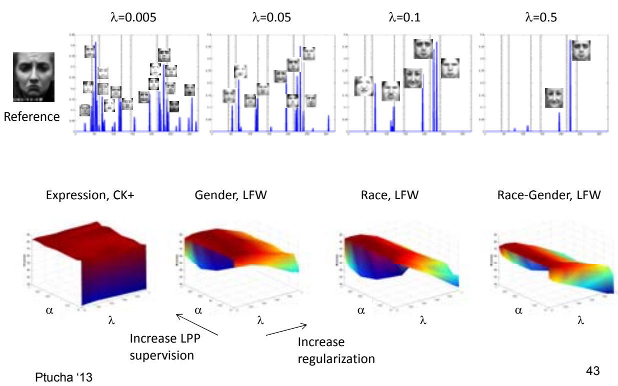
Coefficient Contamination

- Applying the reconstruction error is not a straightforward process for natural images.
- For example, facial identity of the person is often confused with facial expression.
- The usage of semi-supervised manifold learning encourages clustering of sample images in accordance with classification labels.



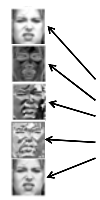


Supervision & Regularization



Region and Pixel Processing

- It is quite conceivable that different regions of the face [Kumar '08] may benefit from different types of pixel processing.
  - Each pixel processing↔facial region combination is a valid feature input to the statistical inference model.

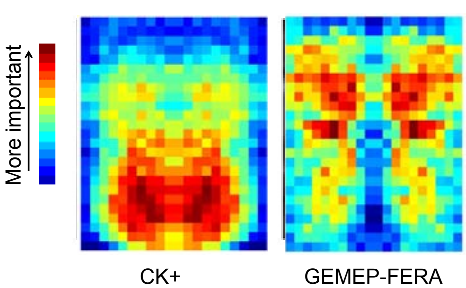


	Fullimg	Face	Mustache	Mouth	EyeReg
Lum	91.4	88.5	82.1	**83.3**	61.0
Mag	88.7	**92.0**	80.1	76.7	**65.3**
Phase	88.9	87.8	70.4	75.4	61.4
LBP	87.7	85 8	63.1	71.1	46.4
Gabor	**91.9**	87.8	**85.5**	79.1	63.9

MSR accuracies on CK expression dataset

Posed vs. Natural Datasets

MSR enables evaluation of any region of the face

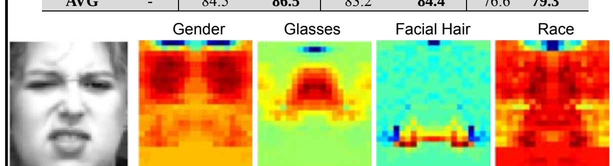


*Correlates well with [Pfister ICCV2011]

MSR Used On Other Facial Attributes

LFW Classification Accuracy

	No. class	SVM no occl.	MSR no occl.	SVM mouth occl.	MSR mouth occl.	SVM eye occl.	MSR eye occl.
Gender	2	89.6	**90.8**	89.8	**90.3**	80.5	**80.8**
Glasses	4	85.0	**87.9**	84.3	**85.0**	71.8	**79.6**
Hair	7	86.9	**87.7**	80.8	**85.6**	87.3	**87.4**
Race	5	85.1	**87.5**	**85.0**	84.3	78.7	**82.0**
Mixed	10	75.9	**78.5**	76.2	**76.6**	64.6	**66.5**
AVG	-	84.5	**86.5**	83.2	**84.4**	76.6	**79.3**



Optimization of Dimensionality Reduction and Sparse Representations

- Sparsity Preserving Projections [Qiao'09] uses (unsupervised) sparse coefficients instead of Laplacian for dimensionality reduction
- Global SR Projections [Lai '09], Discriminative Sparse Coding [Zang '11], and Graph Regularized Sparse Coding [Zheng '11] create variations of joint objective function (DR and SR)
- Supervised LPP [Cai '11] modifies LPP to have (unsupervised) Laplacian and (supervised) LDA properties.
- LC-KSVD [Jiang '11] forces (unsupervised) sparse terms to be (supervised) discriminative and jointly learns a (supervised) classifier

LGE-KSVD

- Each of the previous methods introduce a new dimensionality reduction technique or a new SR technique.
- What lacks is a unified approach that optimizes dimensionality reduction projection matrix *U* with dictionary Φ, and sparse coefficients $\hat{a}$.
- The next few slides will present such a method called LGE-KSVD, for the optimization and infusion of Linear extension of Graph Embedding with K-SVD dictionary learning.
  - Note: LGE is a broader category of linear dimensionality reduction methods which use adjacency matrix *W* to describe neighbor to neighbor topology (includes LDA, LPP, and NPE).

LGE-KSVD

- Classification frameworks based on SR concepts have been found to suffer from:
  1. Coefficient contamination that compromises classification accuracy; and
  2. Computational inefficiencies due to high dimensional features and large dictionaries.
- LGE-KSVD uses:
  - Semi-supervised dimensionality reduction to address both limitations.
  - K-SVD dictionary learning to not only make the dictionaries more efficient, but yield higher classification accuracies.

K-SVD

- K-SVD [Aharon '06] was introduced as a means to learn an over-complete but small dictionary.

$$\{\hat{\Phi}, \hat{a}\} = \min \|x - \Phi a\|_2^2 \quad s.t. \|a\|_0 \le \delta$$

- K-SVD is an iterative technique, where at each iteration, training samples are first sparsely coded using the current dictionary estimate, and then dictionary elements are updated one at a time while keeping others fixed.
- Each new dictionary element is a linear combination of training samples.
- [Rubinstein '08] implemented an efficient implementation of K-SVD using Batch Orthogonal Matching Pursuit (http://www.cs.technion.ac.il/~ronrubin/software.html)

Classification of K-SVD Sparse Coefficients

- Because dictionary elements from K-SVD are a linear combination of input samples, we cannot use the minimum reconstruction error.
- Alternatively we can pass SR coefficients into any regression or machine learning classifier.
- Define *H* as ground truth (GT) matrix, $H \in \mathbf{R}^{k \times n}$.
  - Each column of *H* corresponds to a GT sample. The k^{th} position is 1 if y_i belongs to class k_j, otherwise 0.
- Coefficients a from each training sample are stored in matrix *A*, $A \in \mathbf{R}^{m \times n}$.
- Then solve for coefficient transformation matrix C.

$$\hat{C} = \min \left\| H - C^T A \right\|_2^2 \quad \Rightarrow \quad C = \left(A\ A^T\right)^{-1} A\ H^T$$

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LGE-KSVD Objective Function

- Combining LGE dimensionality reduction with K-SVD minimization functions, we get:

$$\left\{\hat{U}, \hat{\Phi}, \hat{a}\right\} = \min\left\{ \left\| X^T U - \Phi a \right\|_2^2 + \frac{U^T X\ L\ X^T U}{U^T X\ D\ X^T U} \right\} \quad s.t. \left\| a \right\|_0 \le \delta$$

X: input data
U: dim. reduc. matrix
Φ: dictionary
a: sparse coeffs

K-SVD in low dimensional space

LGE dimensionality reduction objective function.

- The above equation is neither directly solvable nor convex.

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LGE-KSVD Objective Function

$$\left\{\hat{U}, \hat{\Phi}, \hat{a}\right\} = \min\left\{ \left\| X^T U - \Phi a \right\|_2^2 + \frac{U^T X\ L\ X^T U}{U^T X\ D\ X^T U} \right\} \quad s.t. \left\| a \right\|_0 \le \delta$$

- We learn a dictionary of *m* atoms, $m \le n$.
- It can be shown that there is an implicit transformation *T*, $\Phi = T X^T U$, where the rank of *T* is greater than the rank of *U*.
- The solution is to use K-SVD to iteratively solve for a, then Φ; then wrap this entire procedure with an update procedure on *U*.

$$\hat{U} = \min \left\| X^T U - A^T \Phi^T \right\|_2^2 \quad \Rightarrow \quad U = \left(X\ X^T\right)^{-1} X\ A^T \Phi^T$$

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Training Procedure for LGE-KSVD

WHILE ε has not converged or $\varepsilon > \tau$
  IF firstIteration
    1a. Calculate *U* using LGE
    1b. Calculate *U* using $U=(XX^T)^{-1}XA^T\Phi^T$
  ENDIF
  2. Calculate low dimensional samples $Y^T=X^TU$
  3. Initializes the *m* samples of Φ randomly from the *n* low dimensional training samples
  4. Calculate $\{A,\Phi\}$ using modified K-SVD, substituting *Y* for *X*.
  5. Calculate *C* using $C=(AA^T)^{-1}AH^T$
  6. Calculate verification set error, $\varepsilon = ||H - C^TA||_2^2$
ENDWHILE

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Testing Procedure for LGE-KSVD

- Given a test sample *x*, along with *U*, Φ, and *C*:

1. Calculate low dimensional sample $y=x^T U$.
2. Use Φ and *y* to calculate sparse coefficients a, using a pursuit* algorithm.
3. Use *C* along with a to estimate class label vector $l \in \mathbf{R}^{k x 1}$ where the maximum value of l is used as a class predictor. $\hat{l} = \max_{i=1:k}\left(l = C^T \hat{a}\right)$

*such as SLEP, http://www.public.asu.edu/~jye02/Software/SLEP

Modified K-SVD

- K-SVD enforces sparsity by fixing the support of each atom in the iteration process to a subset of training samples.
- The addition of supervision injects classification smarts into K-SVD, but still fixes atom support.
- We propose to use semi-supervised LGE adjacency matrix *W* to regulate the support of each dictionary element.

Modified K-SVD

- The support of each dictionary element *j* may:
  - Expand: Modify the support of element *j* by adding (union) all training entries *similar* to element *j*.
  - Contract: Modify the support of element *j* by removing (intersection) training entries *not similar* to element *j*.
  - Redefine: Set the support of element *j* to be only training samples *similar* to element *j*.
  - Fixed: Maintain the support of element *j*, as in the K-SVD algorithm.
- *Similar* is defined in terms of the LGE adjacency matrix

Results: CK+ Expression Dataset

7 static facial expressions, 68 AAM points, 164 training and 163 testing samples

Method	*d* (dimension)	*m* (# dictionary atoms)	*% Accuracy*
PCA	62	-	82.2
LDA	6	-	89.6
LPP	62	-	83.4
NPE	24	-	80.4
SPP	48	-	87.7
K-SVD	136	63	79.1
LC-KSVD1	136	63	79.1
LC-KSVD2	136	63	75.5
SRC	136	164	43.6
MSR	62	164	75.5
LGE based K-SVD	62	63	**92.0**

Results: CK+ Expression Dataset

7 static facial expressions, 60x51 images, 164 training and 163 testing samples

Method	d (dimension)	m (# dictionary atoms)	% Accuracy
PCA	162	-	82.8
LDA	6	-	**86.5**
LPP	163	-	84.7
NPE	71	-	84.0
SPP	80	-	77.9
K-SVD	3060	63	84.0
LC-KSVD1	3060	63	85.9
LC-KSVD2	3060	63	84.7
SRC	500	164	71.8
MSR	163	164	79.1
LGE based K-SVD	163	63	**86.5**

Results: YaleB Recognition Dataset

38 subjects, 192x168 static images reduced to 504 dimensions via random projections, 1216 training and 1198 testing samples

Method	d (dimension)	m (# dictionary atoms)	% Accuracy
PCA	477	-	89.1
LDA	37	-	90.3
LPP	477	-	89.3
NPE	271	-	91.2
SPP	288	-	88.7
K-SVD	504	570	93.2
LC-KSVD1	504	570	93.7
LC-KSVD2	504	570	93.4
SRC	504	1216	86.1
MSR	477	1216	**96.5**
LGE based K-SVD	477	570	95.3

Results: GEMEP-FERA Emotion

5 class, two 24x20 MHI static images per video, 155 training and 134 testing samples

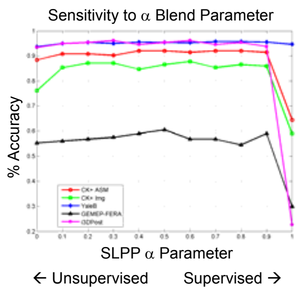
Method	d (dimension)	m (# dictionary atoms)	% Accuracy
PCA	154	-	55.2
LDA	4	-	55.2
LPP	154	-	55.2
NPE	66	-	56.7
SPP	75	-	52.2
K-SVD	1920	75	51.5
LC-KSVD1	1920	75	53.7
LC-KSVD2	1920	75	51.5
SRC	500	155	57.5
MSR	154	155	56.0
LGE based K-SVD	154	75	**60.5**

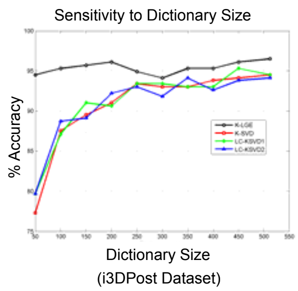
Results: i3DPost Multi-View Activity

12 class, 125 MHI sequences per video, PCA reduced 767 dimensions per video, 512 training and 256 testing samples

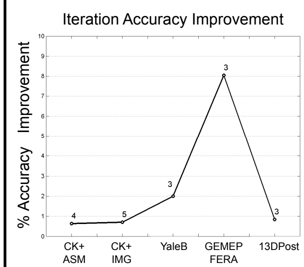
Method	d (dimension)	m (# dictionary atoms)	% Accuracy
PCA	510	-	94.9
LDA	510	-	94.5
LPP	510	-	**96.1**
NPE	224	-	94.9
SPP	241	-	91.0
K-SVD	767	450	94.1
LC-KSVD1	767	450	95.3
LC-KSVD2	767	450	93.8
SRC	767	512	88.7
MSR	510	512	95.3
LGE based K-SVD	510	450	**96.1**

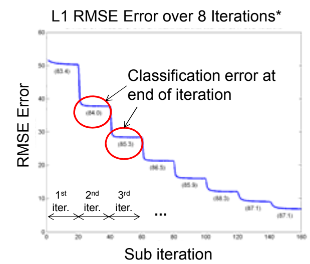
LGE-KSVD Analysis





LGE-KSVD Analysis



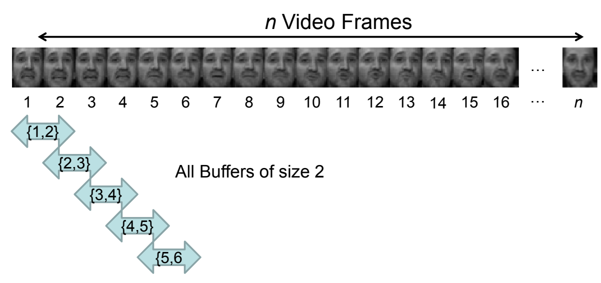


*Each iteration has 20 K-SVD sub iterations
CK+ w/ images

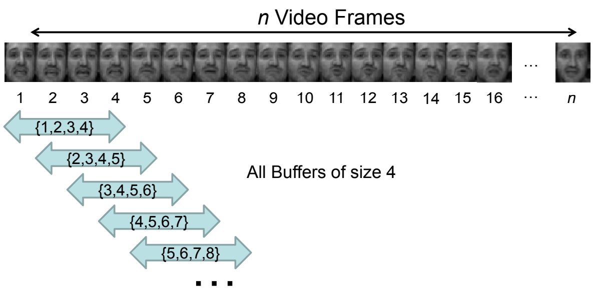
Temporal Processing

- Communication between humans naturally contains temporal signature.
  - Rolling of eyes, waving of hand, wink, etc.
- Previous studies adopted both sparse and dense optical flow techniques and contrast to static methods.
- Facial expressions and gestures can occur at any point in time and are variable in length.
- We define sliding temporal windows, W^{θ}_{l}, each of duration θ frames, $l=1..m$ sliding windows.

Examine Video In Variable Size Rolling Frame Buffers



Examine Video In Variable Size Rolling Frame Buffers



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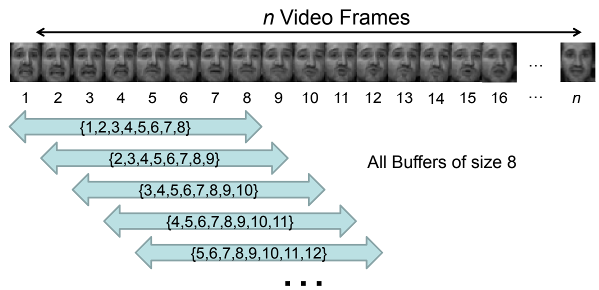
Examine Video In Variable Size Rolling Frame Buffers



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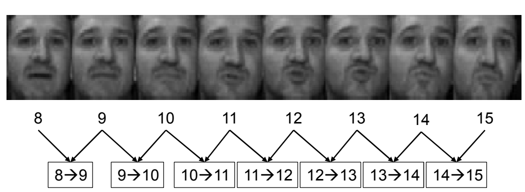
Analysis Example

- Lets say, we are looking at window widths of 8.
- Our first position center is frame 12.
- We then look at 7 motion trajectories:



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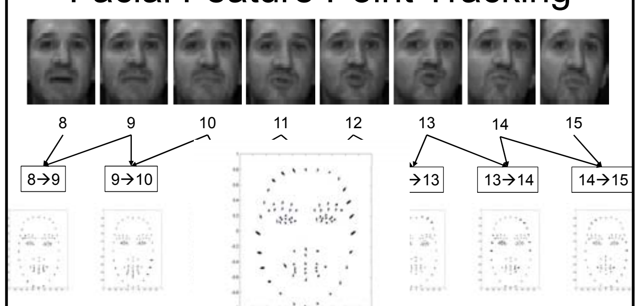
Facial Feature Point Tracking



Similarly, can compute point tracking from current frame the mean frame.

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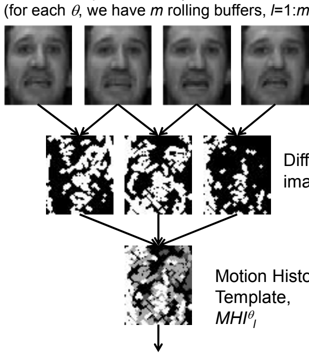
Motion History Images

[Bobick ‘01][Koelstra ‘10]

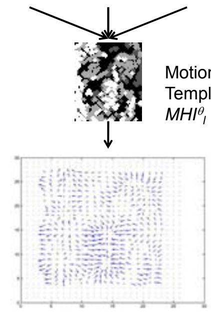
Example buffer W^{θ}_{l} of size θ=4
(for each θ, we have m rolling buffers, l=1:m)



Difference images

Motion History Template, MHI^{θ}_{l}

Motion History Images (Cont’d)



Motion History Template, MHI^{θ}_{l}

Pixels point towards recent movement

Δx and Δy of each vector passed into classifier

Summary

- Face and gesture understanding problems can be reliably solved in unconstrained scenes using SRs.
- The usage of semi-supervised LPP before SR clusters by classification task, avoiding coefficient contamination.
- The usage of K-SVD dictionary learning makes the dictionaries more compact and results in higher classification accuracies.
- If the training dictionary is not over complete, SR methods have trouble generalizing test samples from training dictionary exemplars.

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