Back projection – 2D points to 3D

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Let,

K = Intrinsic matrix of the camera

$[R \mid t]$= Extrinsic matrix of the camera

C = Camera center in world co-ordinate system

P = 3D point on a plane with normal $\hat{n}$ expressed in the world co-ordinate system (3D)

c = Projection of point P on the image plane (2D)

w = scaling factor (used in projection of 3D point to 2D)

$\hat{n}$ = $[a\ b\ c]$ represent the normal to the plane

(Equation of the plane = $ax + by + cz + d = 0$)

$$wp = K\ [R\ |t]\ P$$

$$= K\ [RP + t]$$

$$= KR\ [P - C] \quad \text{(Note: Camera center,} C = -R^{-1}t\text{)} \text{ ---------- (1)}$$

Using (1),

$$w[KR]^{-1}p + C = P \text{ ---------- (2)}$$

P is a point on the plane, $=> \hat{n}.P + d = 0$

$$=> \hat{n}\,.(P - C) + d + \hat{n}C = 0$$

$$=> \hat{n}.(w[KR]^{-1}p) + d + \hat{n}C = 0$$

$$=> w = \frac{(-d - \hat{n}C)}{\hat{n}.([KR]^{-1}p)} \text{ ---------- (3)}$$

Substituting (3) in (2),

$$\boldsymbol{P} = \frac{(-\boldsymbol{d} - \hat{\boldsymbol{n}}\boldsymbol{C})}{\hat{\boldsymbol{n}}.\ ([\boldsymbol{KR}]^{-1}\boldsymbol{p})}\ [\boldsymbol{KR}]^{-1}\boldsymbol{p} + \boldsymbol{C}$$