Department of Electrical and Computer Engineering
Department of Radiology

Sparsity-Based Deconvolution: A new data-driven method for low-dose perfusion CT quantification

Ruogu Fang, Tsuhan Chen — Cornell University
Pina Sanelli, Ashish Raj — Weill Cornell Medical College

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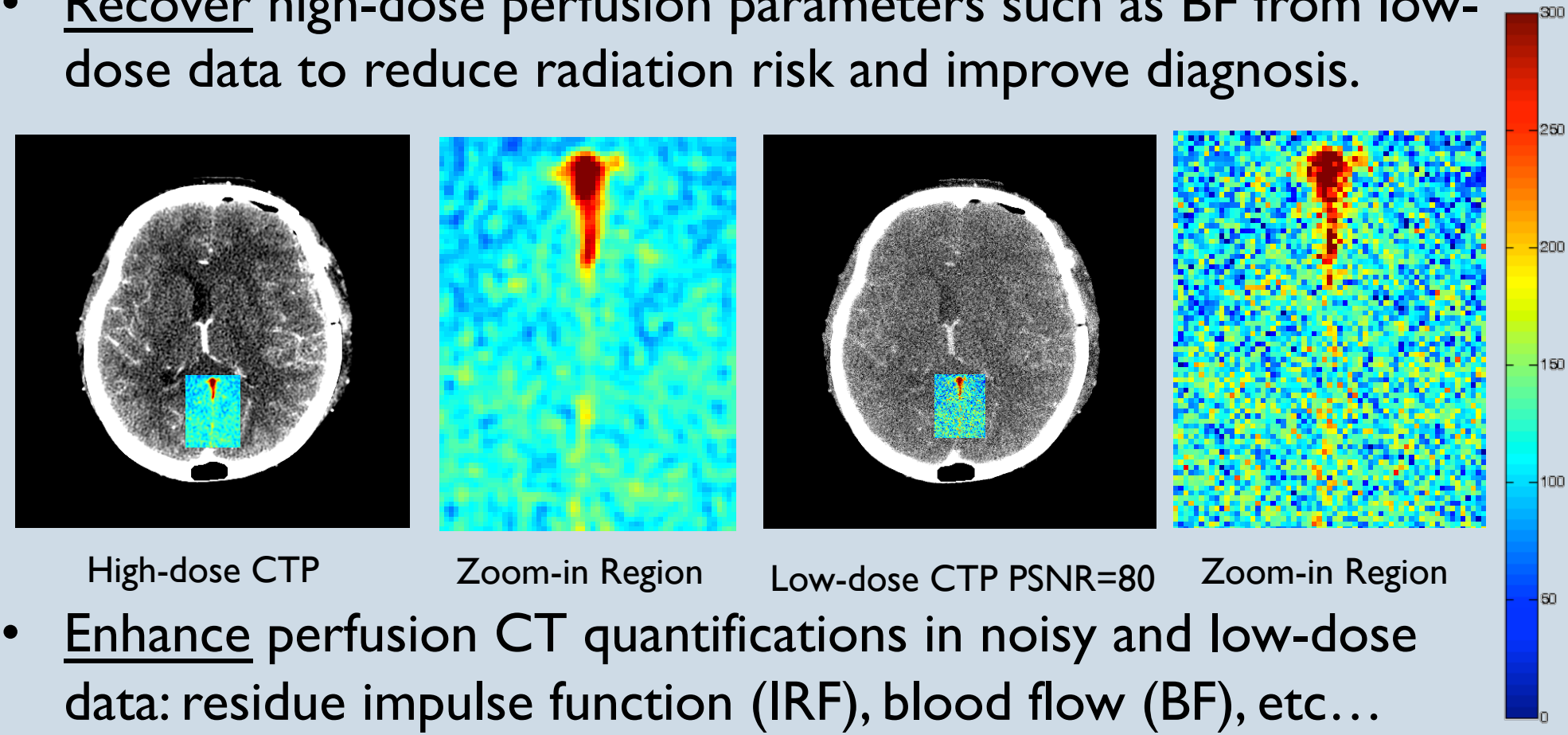
Motivation for Improving Low-dose Perfusion CT

- <u>Recover</u> high-dose perfusion parameters such as BF from low-dose data to reduce radiation risk and improve diagnosis.

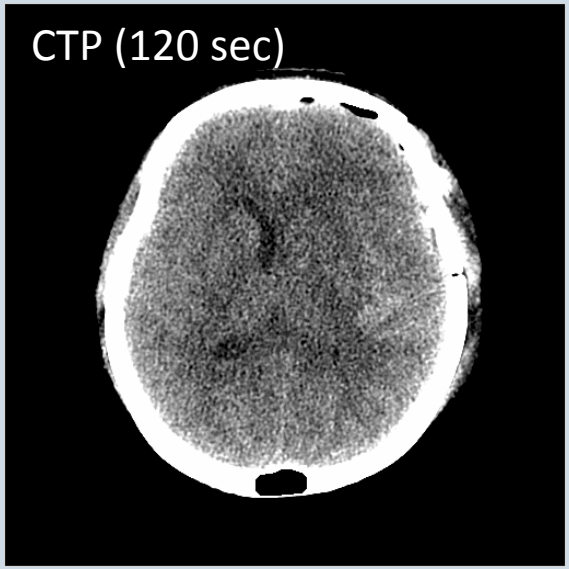


High-dose CTP | Zoom-in Region | Low-dose CTP PSNR=80 | Zoom-in Region

- <u>Enhance</u> perfusion CT quantifications in noisy and low-dose data: residue impulse function (IRF), blood flow (BF), etc…

Priors in perfusion CT

- Regularization priors used to quantify perfusion parameters:
  - Temporal convolution
  - Learning from data



Priors in perfusion CT

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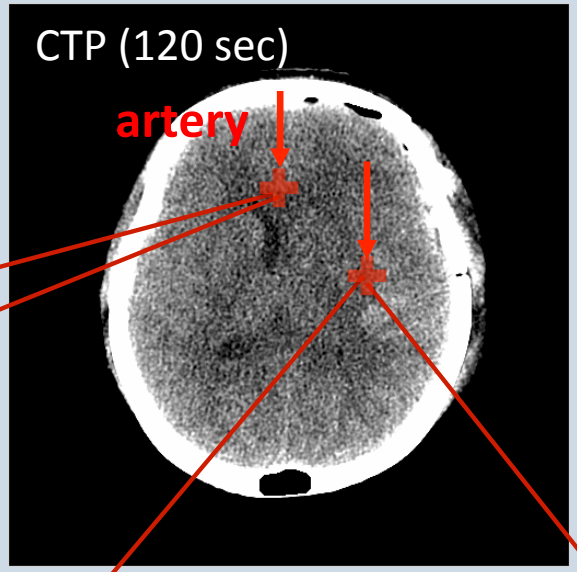
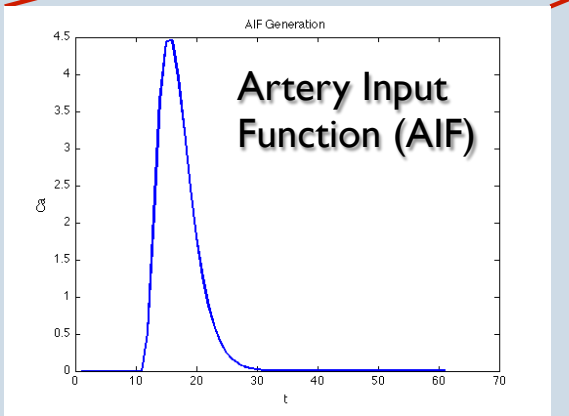


Indicator dilution theory :



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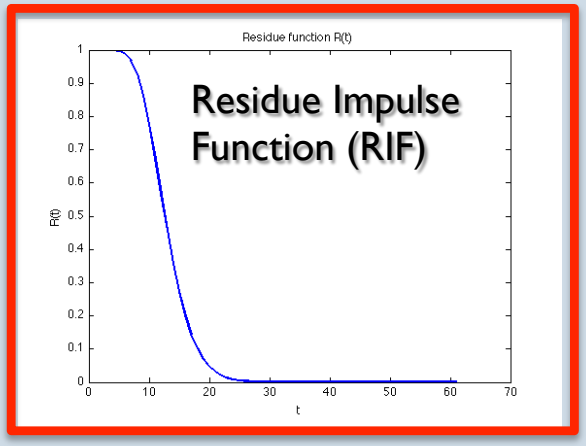

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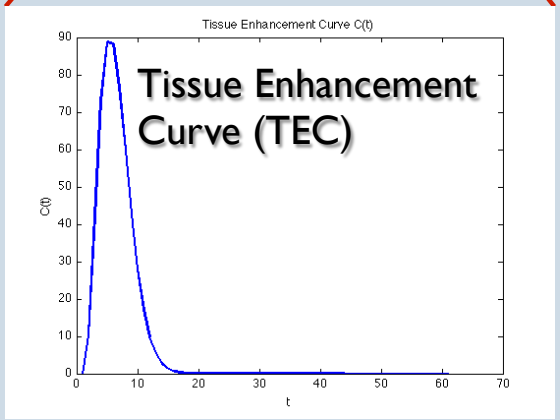
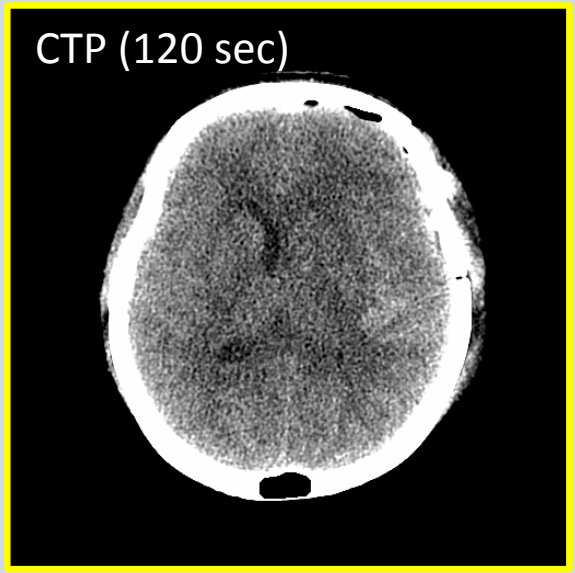
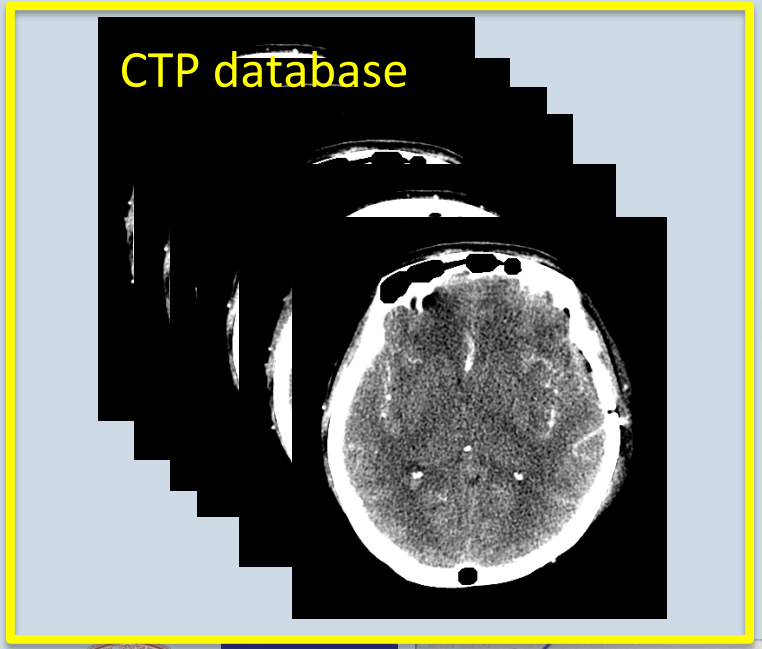
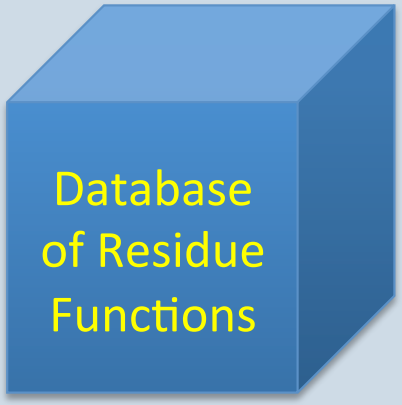


Priors in perfusion CT

- Regularization priors used to quantify perfusion parameters:
  - Temporal convolution
  - Learning from data

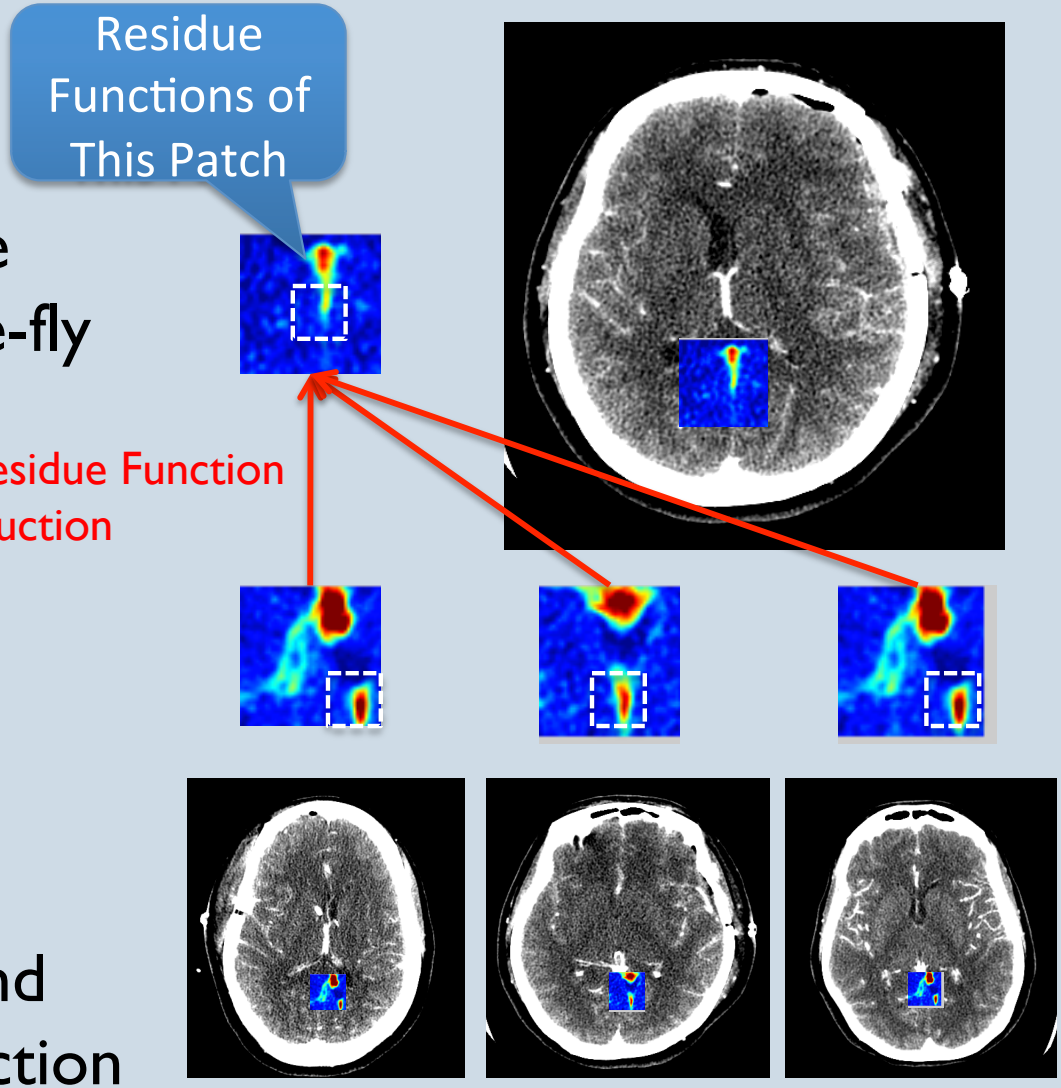






Concept of Sparse Residue Representation (SRR)

- Data-driven
  - Perfusion parameters are learned from data on-the-fly through sparse representation
- Advantages
  - No assumption on any parametric model of perfusion curves
  - Highly robust to noise and yield accurate reconstruction

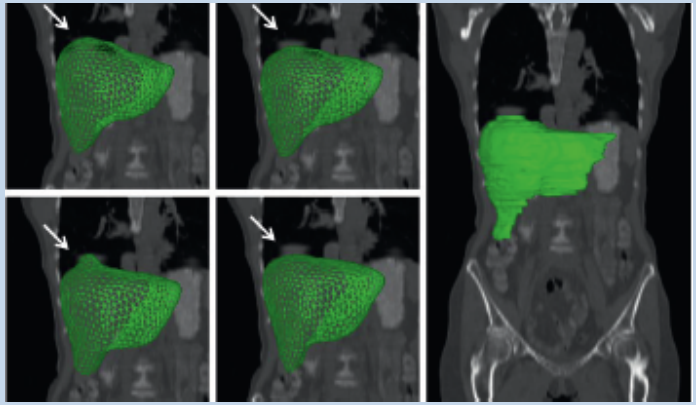


Related Work: applications of sparse representation

- Image Denoising: sparse and redundant representation

- M. Elad. IEEE Trans Image Processing 2006

- Deformable Segmentation: sparse shape representation



- S. Zhang. Medical Image Analysis 2011

<u>Our contribution:</u>

Sparsity prior in residue functions for CT perfusion quantification

Sparse Residue Representation: Our model

$$J = \left\| C_v - C_a R \right\|_2^2 + \mu \left\| x \right\|_0 = \left\| C_v - C_a D x \right\|_2^2 + \mu \left\| x \right\|_0$$

- R: residue impulse function (RIF)
- D: dictionary learned from training data
- x: sparse coefficients for linear combination
- Cv: tissue enhancement curve (TEC)
- Ca: artery input function (AIF)
- μ : sparsity regularization parameter

Sparse Residue Representation: Our model

$$J = \boxed{\left\| C_v - C_a R \right\|_2^2} + \mu \left\| x \right\|_0 = \left\| C_v - C_a D x \right\|_2^2 + \mu \left\| x \right\|_0$$

Temporal Convolution

- R: residue impulse function (RIF)
- D: dictionary learned from training data
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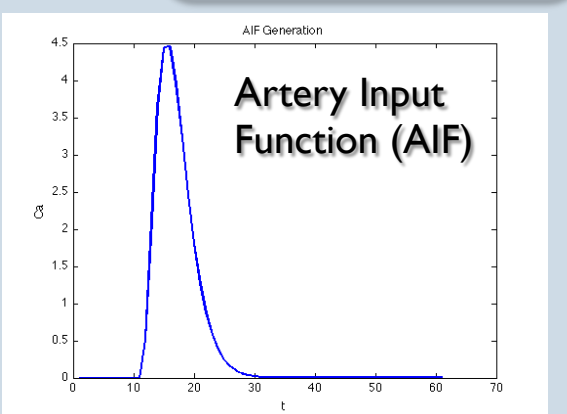

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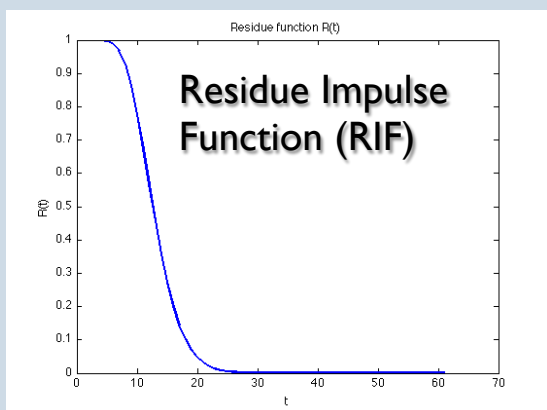

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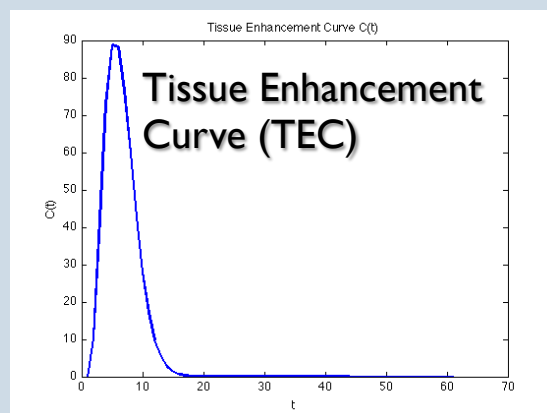
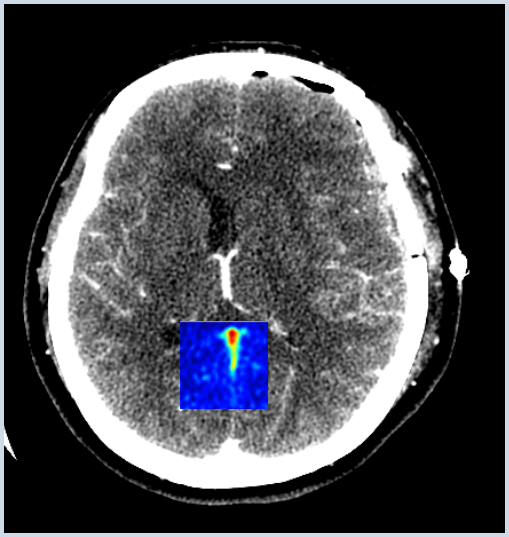


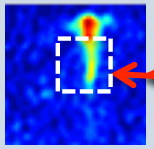
Sparse Residue Representation: Our model

$$J = \left\| C_v - C_a R \right\|_2^2 + \mu \left\| x \right\|_0 = \left\| C_v - C_a D x \right\|_2^2 + \mu \left\| x \right\|_0$$

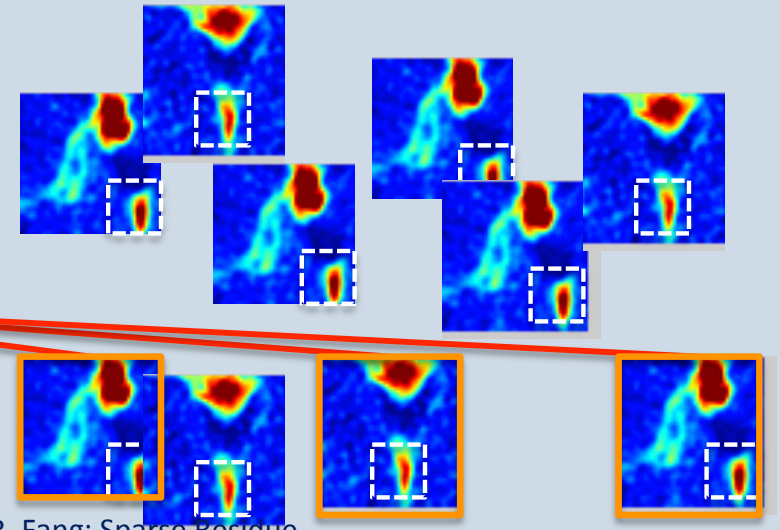
Sparsity Constraint

- R: residue impulse function (RIF)
- D: dictionary learned from training data
- x: sparse coefficients for linear combination
- Cv: tissue enhancement curve (TEC)
- Ca: artery input function (AIF)
- μ : sparsity regularization parameter





Sparse Residue Function Reconstruction



Sparse Residue Representation: Our model

$$J = \|C_v - C_a R\|_2^2 + \mu \|x\|_0 = \|C_v - C_a D x\|_2^2 + \mu \|x\|_1$$

L_1 Relaxation

- R: residue impulse function (RIF)
- D: dictionary learned from training data
- x: sparse coefficients for linear combination
- Cv: tissue enhancement curve (TEC)
- Ca: artery input function (AIF)
- μ : sparsity regularization parameter

Solve by SLEP: Sparse Learning with Efficient Projections.

Baseline: cTSVD

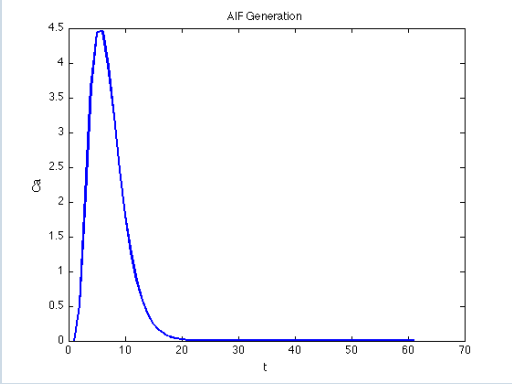
- cTSVD: circulant Truncated Singular Value Decomposition
  - Most commonly used deconvolution method
  - Truncate the small singular values to zero to remove oscillation
  - Sensitive to varying contrast (bias field) and noise

Numerical Simulation

- AIF Generation: Gamma-variant function

$$C_a(t) = \begin{cases} 0 & t \le t_0 \\ a(t-t_0)^b e^{-(t-t_0)/c} & t > t_0 \end{cases}$$

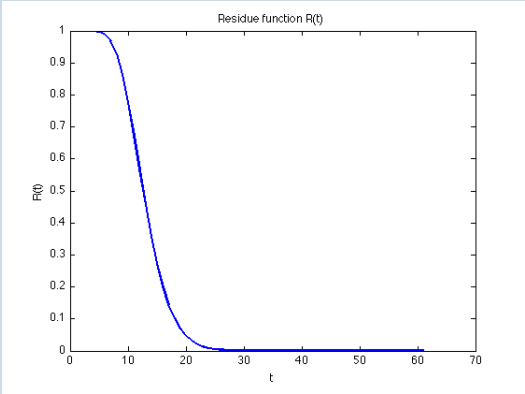
a=1 b=3
c=1.5 t0=1



- Residue Function Generation: Family of gamma distributions

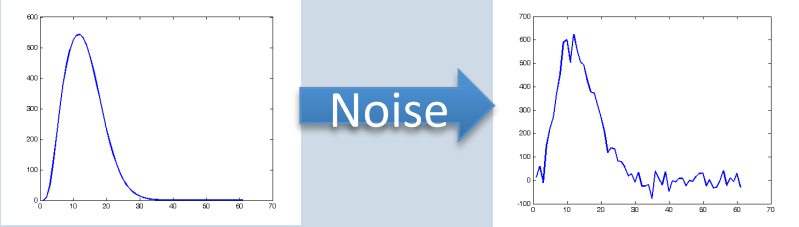
$$h(t;\alpha,\beta) = \frac{1}{\beta^\alpha \Gamma(\alpha)} t^{\alpha-1} e^{-t/\beta} \qquad \alpha,\beta > 0$$

$$R(t) = 1 - \int_0^t h(\tau)d\tau \qquad \beta = BV/(\alpha \cdot BF)$$



- Gaussian Noise Generation:

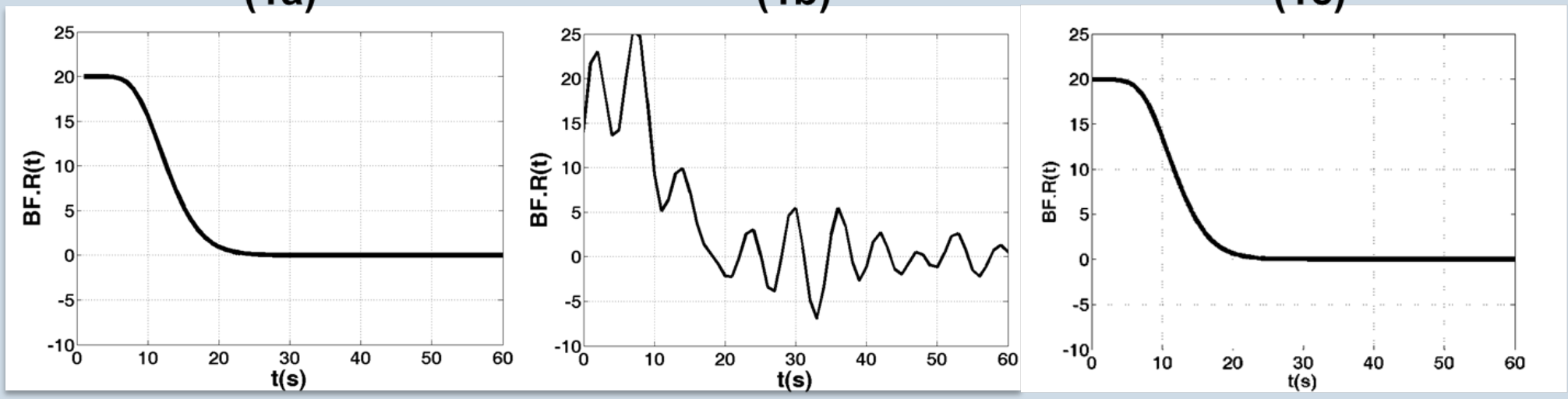
$$\varepsilon \sim N(0,\sigma^2)$$



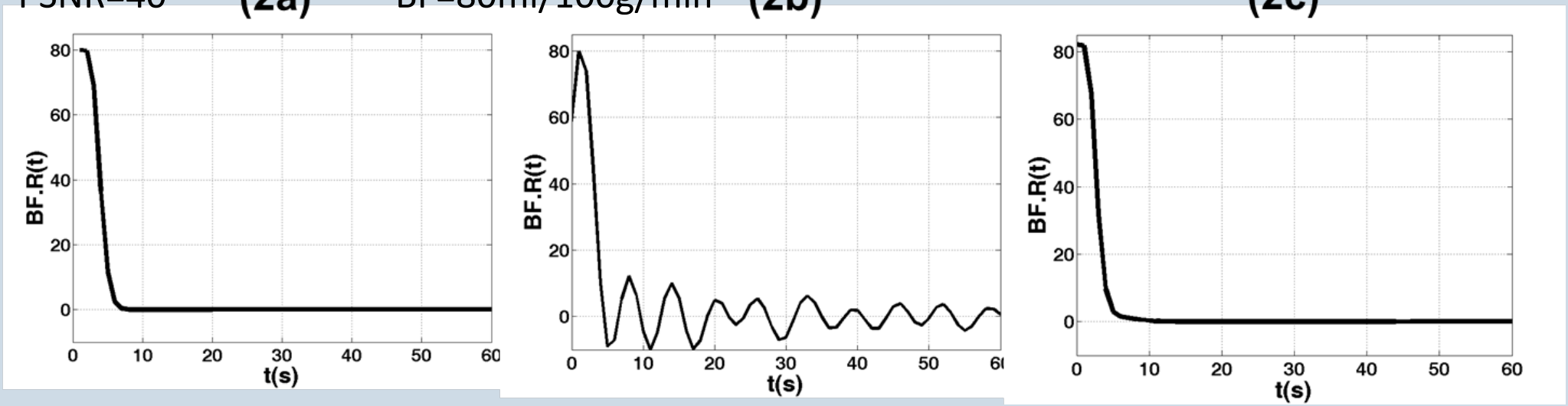
Residue Function Recovery

PSNR=20, BV=4ml/100g, BF=20ml/100g/min

(1a) (1b) (1c)



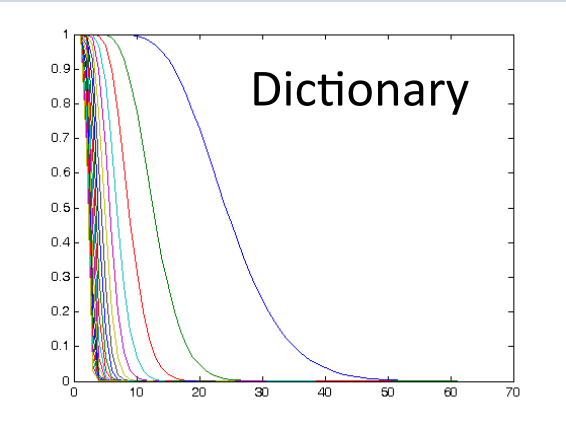
PSNR=40 (2a) BF=80ml/100g/min (2b) (2c)

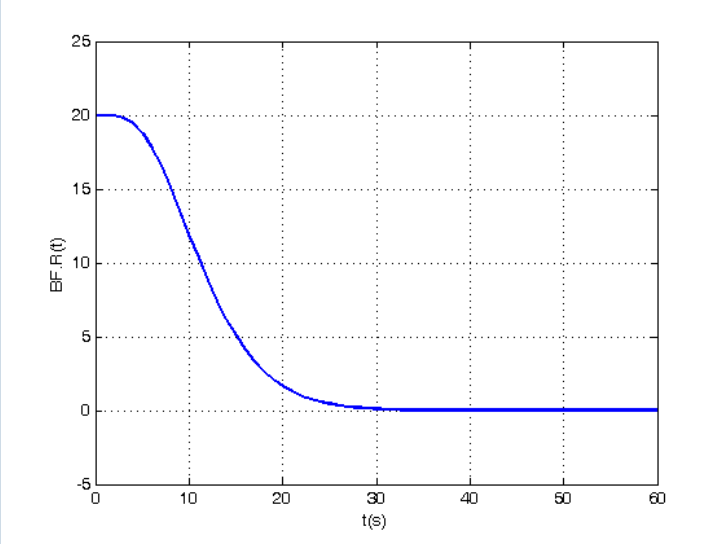


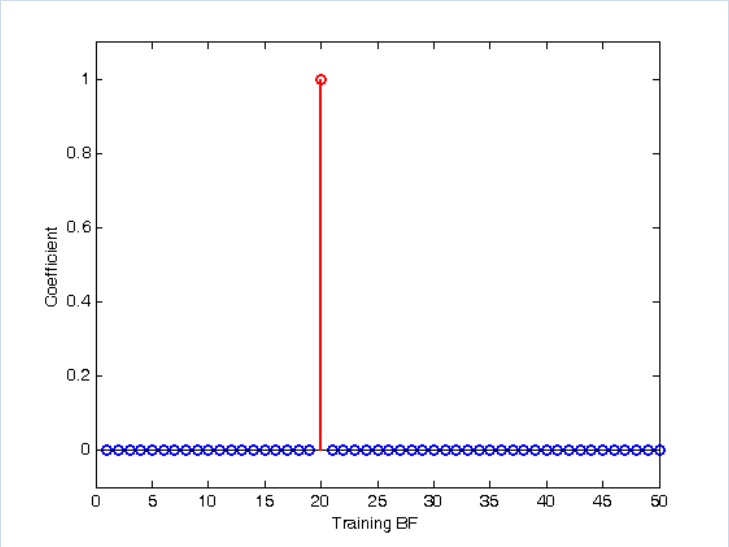
(a) True Residue (b) cTSVD (c) SRR

Sparse Recovery

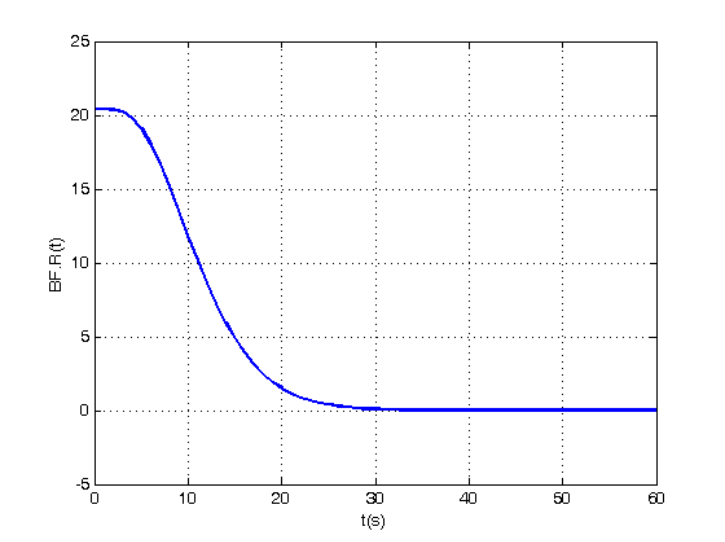
BF in D

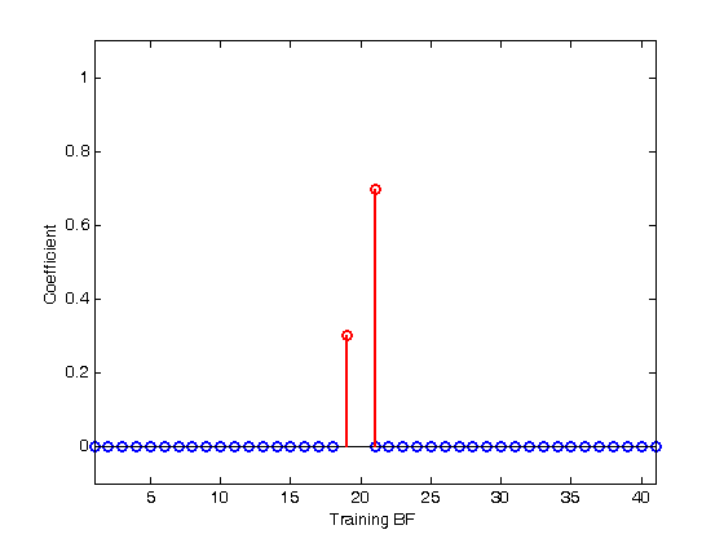




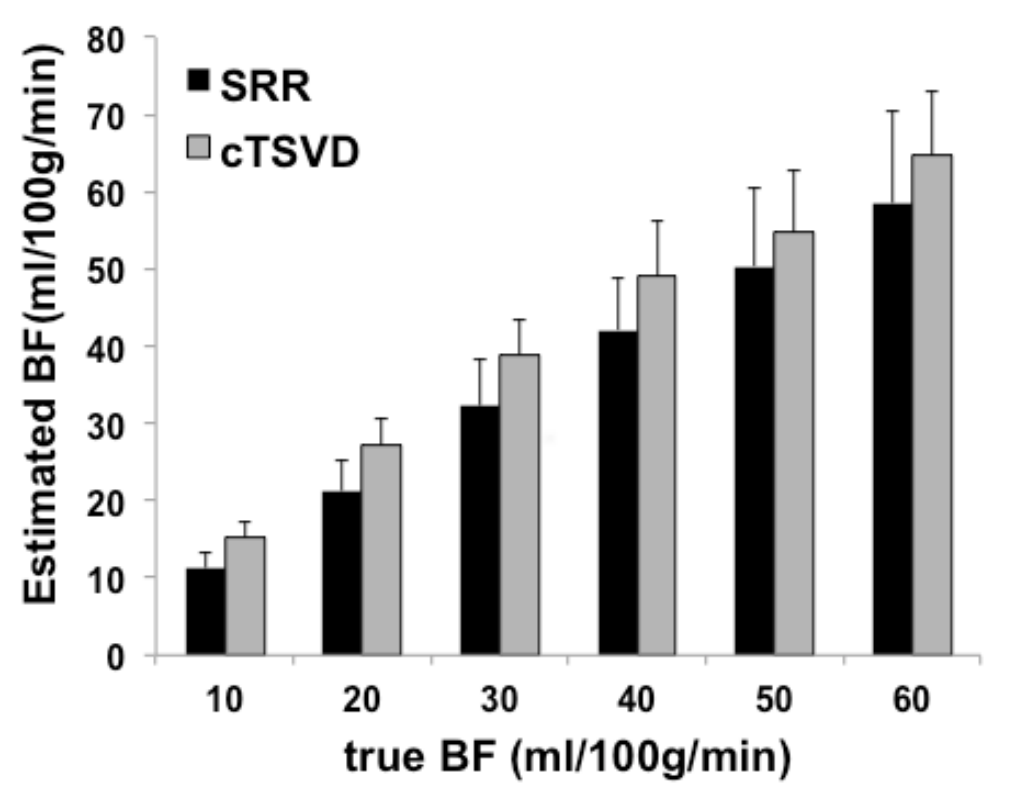


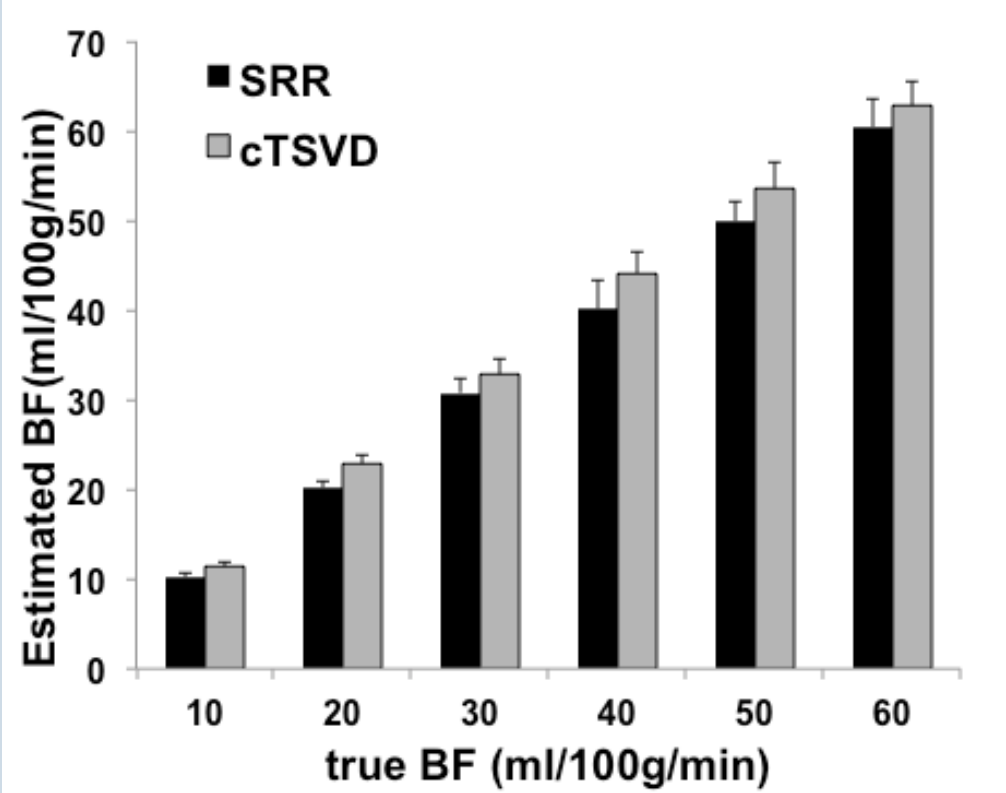
BF NOT in D





BF Estimation

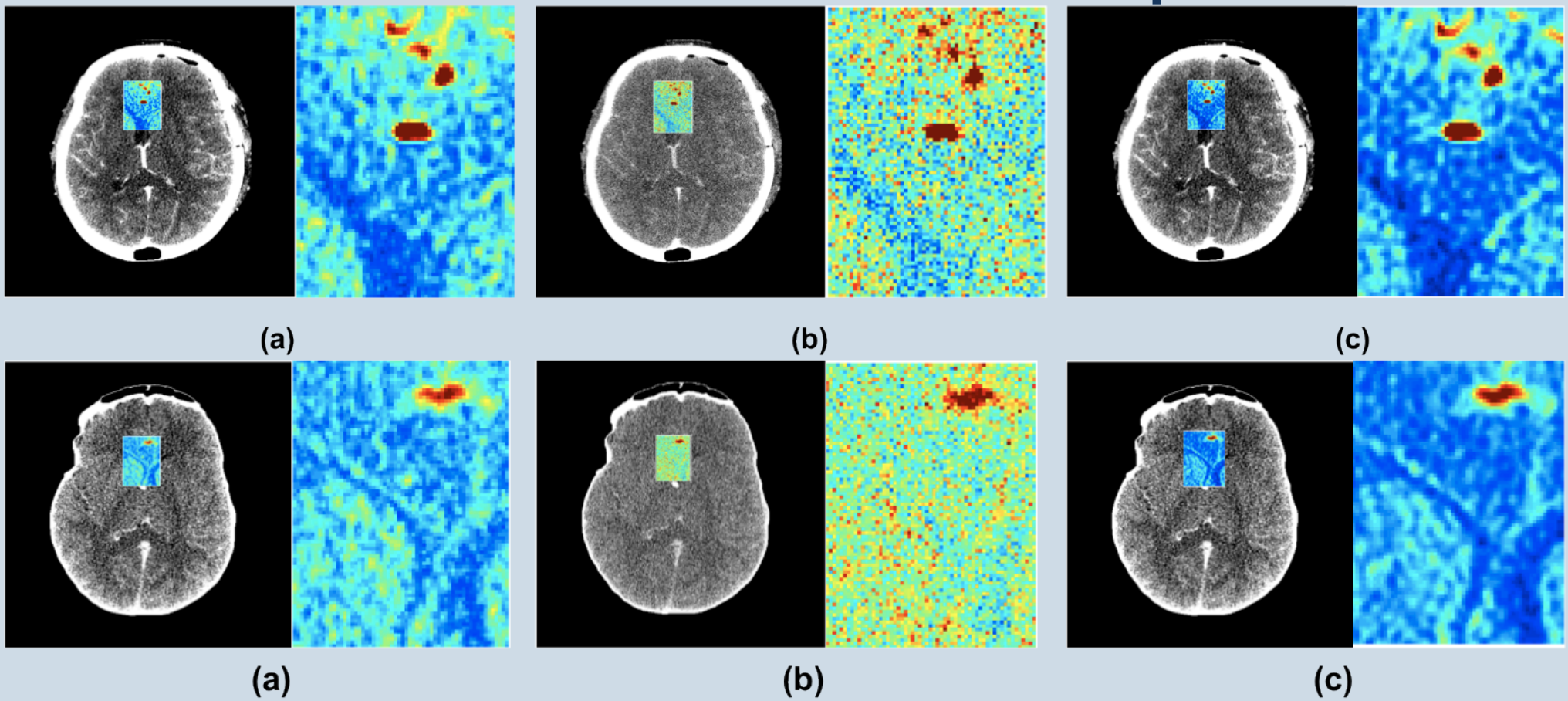




PSNR = 10
SPR: MSE = 2.4974
cTSVD: MSE = 47.1007

PSNR = 40
SPR: MSE = 0.1536
cTSVD: MSE = 9.1072

Clinical Results: BF Maps



BF maps and zoomed-in regions of a vasospasm patient (above row) and normal patient (below row) using (a) high-dose TSVD (b) low-dose TSVD and (c) low-dose sparse residue representation.

Clinical Results: BF Maps

Subjects	Variations		MSE	
	SRR	TSVD	SRR	TSVD
1	23.96	38.06	179.38	745.68
2	26.68	46.40	263.42	545.38
3	28.23	33.99	201.38	865.492
4	18.46	34.29	229.45	715.59

Table 1. BF variations (ml/100g/min) and mean square error (MSE) over certain ROIs estimated by sparse residue representation and TSVD.

Clinical Results: Ischemic Detection

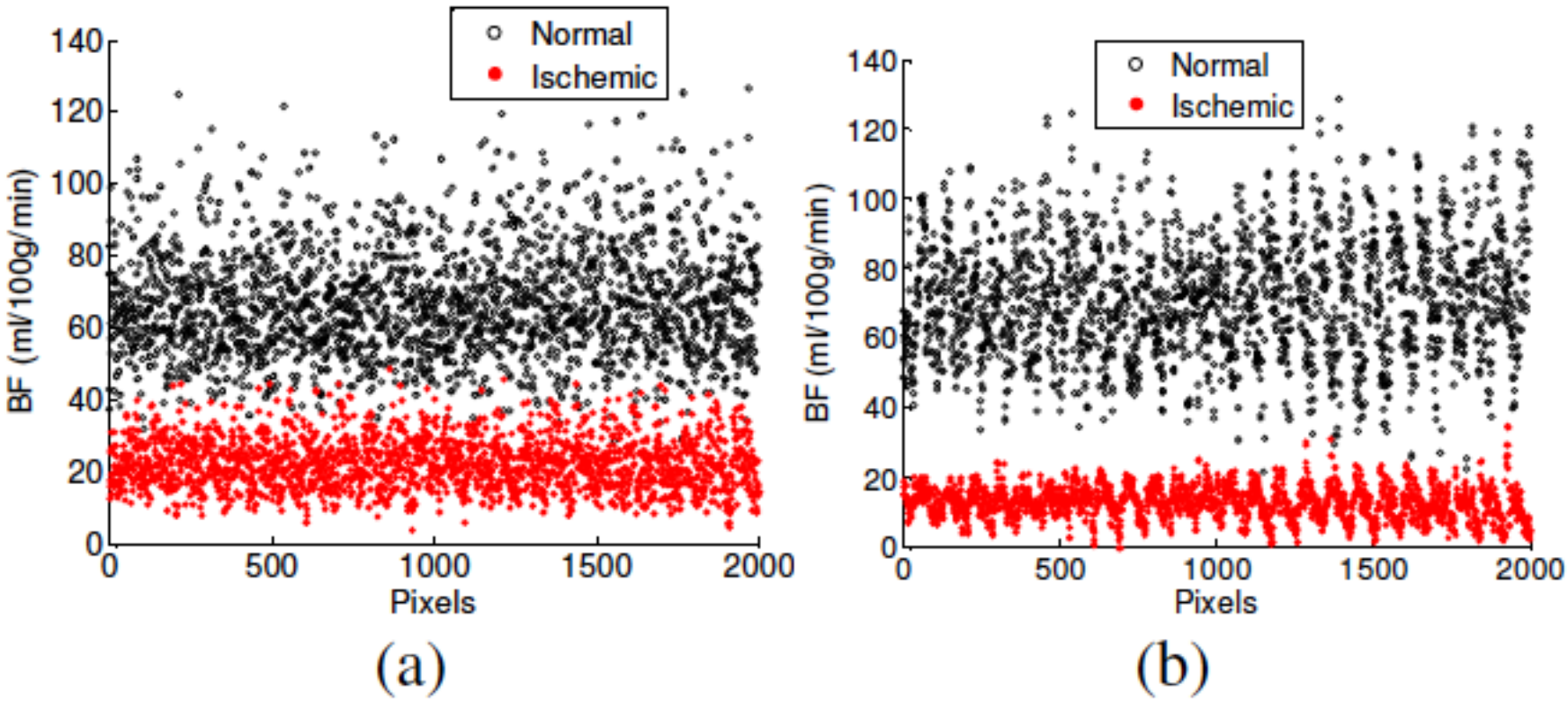
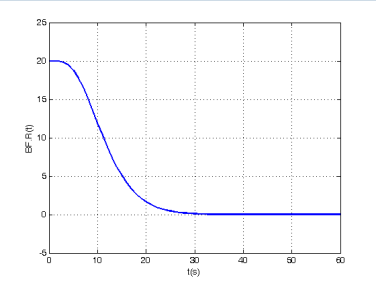
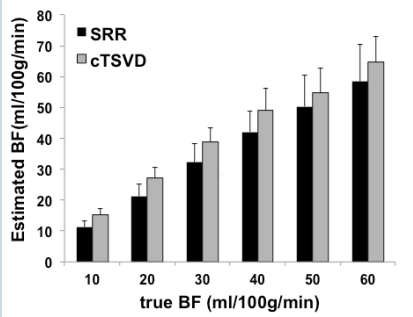


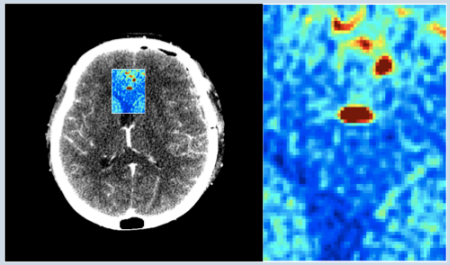
Fig. 4. (a) Two clusters of normal vs. abnormal generated by TSVD method. The distance d between two clusters is 118.08. (b) Two clusters of normal vs. abnormal generated by our sparse residue representation method. The distance d between two clusters is 148.57.

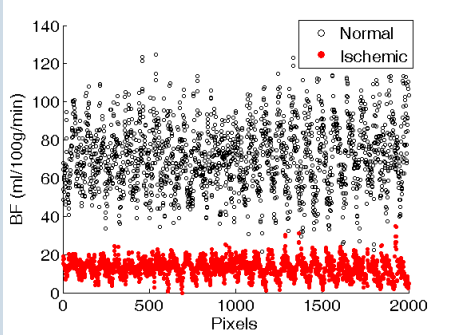
Conclusion

- Sparse residue deconvolution
  - Based on temporal convolution and sparsity prior in terms of residue functions
- Advantages compared with cTSVD
  - Robust against: noise, varying BF, baseline oscillation…
  - Avoid overestimation of BF
  - Improve spatial smoothness of uniform areas
  - Enlarge differences between ischemic and normal tissues









- Thank you for your attention!

- Acknowledgements
  - We would like to thank NINDS, a component of NIH for funding this work within Cornell and Weill Cornell Medical College
  - We also acknowledges funding from NINDS 5K23NS058387-02