Sparsity-Based Deconvolution: A new data-driven method for low-dose perfusion CT quantification

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Motivation for Improving Low-dose Perfusion CT

- **Recover** high-dose perfusion parameters such as BF from low-dose data to reduce radiation risk and improve diagnosis.

- **Enhance** perfusion CT quantifications in noisy and low-dose data: residue impulse function (IRF), blood flow (BF), etc…

![Image of High-dose CTP, Zoom-in Region, Low-dose CTP PSNR=80, and Zoom-in Region](image)
Regularization priors used to quantify perfusion parameters:

- Temporal convolution
- Learning from data
Priors in perfusion CT

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Indicator dilution theory:

CTP (120 sec)
**Priors in perfusion CT**

- Regularization priors used to quantify perfusion parameters:
  - Temporal convolution
  - Learning from data

**CTP (120 sec)**

**Database of Residue Functions**


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Concept of Sparse Residue Representation (SRR)

• Data-driven
  – Perfusion parameters are learned from data on-the-fly through sparse representation

• Advantages
  – No assumption on any parametric model of perfusion curves
  – Highly robust to noise and yield accurate reconstruction
Related Work: applications of sparse representation

- Image Denoising: sparse and redundant representation

- Deformable Segmentation: sparse shape representation
  - S. Zhang. Medical Image Analysis 2011

Our contribution:
Sparsity prior in residue functions for CT perfusion quantification
Sparse Residue Representation: Our model

\[ J = \| C_v - C_a R \|_2^2 + \mu \| x \|_0 = \| C_v - C_a D x \|_2^2 + \mu \| x \|_0 \]

- **R**: residue impulse function (RIF)
- **D**: dictionary learned from training data
- **x**: sparse coefficients for linear combination
- **C_v**: tissue enhancement curve (TEC)
- **C_a**: artery input function (AIF)
- **\( \mu \)**: sparsity regularization parameter
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Sparse Residue Function Reconstruction


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Sparse Residue Representation: Our model

\[ J = \|C_v - C_a R\|_2^2 + \mu \|x\|_0 = \|C_v - C_a D x\|_2^2 + \mu \|x\|_1 \]

- \(R\): residue impulse function (RIF)
- \(D\): dictionary learned from training data
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L₁ Relaxation

Solve by SLEP: Sparse Learning with Efficient Projections.
Baseline: cTSVD

- **cTSVD**: circulant Truncated Singular Value Decomposition
  - Most commonly used deconvolution method
  - Truncate the small singular values to zero to remove oscillation
  - Sensitive to varying contrast (bias field) and noise
Numerical Simulation

- **AIF Generation: Gamma-variant function**
  
  \[ C_a (t) = \begin{cases} 
  0 & t \leq t_0 \\
  a(t - t_0)^b e^{-(t-t_0)/c} & t > t_0 
  \end{cases} \]

  \( a=1 \quad b=3 \quad c=1.5 \quad t_0=1 \)

- **Residue Function Generation: Family of gamma distributions**

  \[ h(t; \alpha, \beta) = \frac{1}{\beta^\alpha \Gamma(\alpha)} t^{\alpha-1} e^{-t/\beta} \quad \alpha, \beta > 0 \]

  \[ R(t) = 1 - \int_0^t h(\tau) d\tau \quad \beta = BV / (\alpha \cdot BF) \]

- **Gaussian Noise Generation:**

  \[ \varepsilon \sim N(0, \sigma^2) \]
Residue Function Recovery

PSNR=20, BV=4ml/100g, BF=20ml/100g/min

(a) True Residue

(b) cTSVD

(c) SRR

PSNR=40

BF=80ml/100g/min


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Sparse Recovery

BF in D

Dictionary

BF NOT in D


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BF Estimation

PSNR = 10
SPR: MSE = 2.4974
cTSVD: MSE = 47.1007

PSNR = 40
SPR: MSE = 0.1536
cTSVD: MSE = 9.1072


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BF maps and zoomed-in regions of a vasospasm patient (above row) and normal patient (below row) using (a) high-dose TSVD (b) low-dose TSVD and (c) low-dose sparse residue representation.
Clinical Results: BF Maps

<table>
<thead>
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<th>Subjects</th>
<th>Variations</th>
<th>MSE</th>
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<tbody>
<tr>
<td></td>
<td>SRR</td>
<td>TSVD</td>
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</tr>
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<tr>
<td>4</td>
<td>18.46</td>
<td>34.29</td>
</tr>
</tbody>
</table>

Table 1. BF variations (ml/100g/min) and mean square error (MSE) over certain ROIs estimated by sparse residue representation and TSVD.
Clinical Results: Ischemic Detection

Fig. 4. (a) Two clusters of normal vs. abnormal generated by TSVD method. The distance $d$ between two clusters is 118.08. (b) Two clusters of normal vs. abnormal generated by our sparse residue representation method. The distance $d$ between two clusters is 148.57.
Conclusion

• Sparse residue deconvolution
  – Based on temporal convolution and sparsity prior in terms of residue functions

• Advantages compared with cTSVD
  – Robust against: noise, varying BF, baseline oscillation...
  – Avoid overestimation of BF
  – Improve spatial smoothness of uniform areas
  – Enlarge differences between ischemic and normal tissues
• Thank you for your attention!

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