Multi-View 3D Reconstruction for Scenes under the Refractive Plane with Known Vertical Direction

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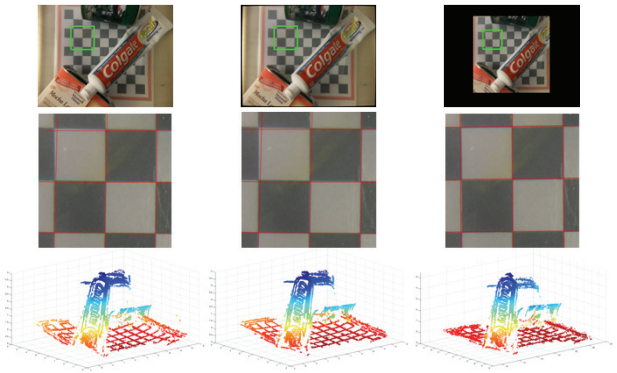
Motivations

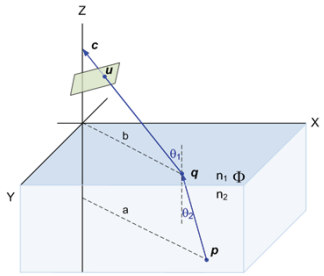
Reformulation of SfM with refractive distortion model



No distortion model | Radial distortion model | Refractive distortion model

Physics

Forward Projection



Refrax ratio: $\lambda = \frac{b}{a}$

Two-step projection:

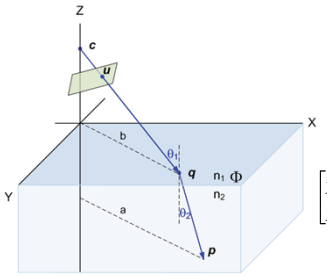
$$\mathbf{q} = \begin{bmatrix} c_x + \lambda(p_x - c_x) \\ c_y + \lambda(p_y - c_y) \\ 0 \end{bmatrix}$$

$$\mathbf{u} \equiv \mathbf{K}(\mathbf{R}\mathbf{q} + \mathbf{t})$$

Solving the refrax ratio with the 4th-order equation:

$$f(\lambda) = Na^4\lambda^4 - 2Na^4\lambda^3 + \left(Na^2 - \frac{p_z^2}{\delta^2} + c_z^2\right)a^2\lambda^2 - 2a^2c_z^2\lambda - a^2c_z^2 = 0$$

Back Projection



$$\mathbf{q} = \mathbf{c} + w\mathbf{R}^\top\mathbf{K}^{-1}\mathbf{u} = \mathbf{c} + w\mathbf{s}$$

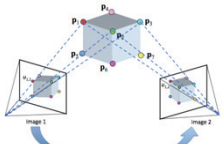
$$\begin{bmatrix} p_x \\ p_y \end{bmatrix} = \begin{bmatrix} c_x \\ c_y \end{bmatrix} - \frac{1}{s_z}\begin{bmatrix} s_x \\ s_y \end{bmatrix}\left(c_z - \frac{p_z}{g(\mathbf{s})}\right)$$

Linear relation between 3D points and camera centers

➡ Useful for triangulation!

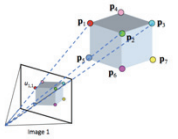
Pose Estimation & Bundle Adjustment

Relative Pose



$[\mathbf{R}_{ver}\mathbf{R}_z \ \ \mathbf{t}]$

Absolute Pose

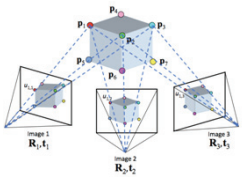


$[\mathbf{R}_{ver}\mathbf{R}_z \ \ \mathbf{t}]$

	Relative Pose	Absolute Pose
Single medium Known vertical direction	3 DOFs (3 trans. ↑ scale + 1 rot.) 5 pts linear sol.	4 DOFs (3 trans. + 1 rot.) 2 pts minimal case sol.
Two mediums **Known vertical direction**	4 DOFs (3 trans. + 1 rot.) 7 pts linear sol.	4 DOFs (3 trans. + 1 rot.) 2 pts minimal case sol.

Known vertical direction
= Known incident angle for each light ray

Bundle Adjustment



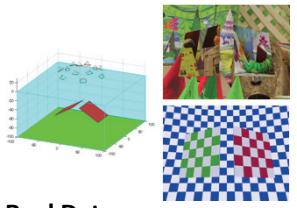
$$(\mathbf{C}^*, \mathbf{P}^*, \mathbf{R}^*) = \underset{\mathbf{C},\mathbf{P},\mathbf{R}}{\operatorname{argmin}} \sum_{i,j} \|\mathbf{u}_{ij} - \hat{\mathbf{u}}_{ij}\|_2^2$$

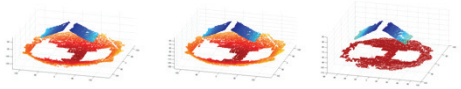
$$\begin{bmatrix} \hat{\mathbf{u}}_{ij} \\ 1 \end{bmatrix} \equiv \mathbf{K}\mathbf{R}(\hat{\mathbf{q}}_{ij} - \hat{\mathbf{c}}_j) = \mathbf{K}\mathbf{R}\begin{bmatrix} \hat{\lambda}_{ij}(\hat{p}_{ix} - \hat{c}_{jx}) \\ \hat{\lambda}_{ij}(\hat{p}_{iy} - \hat{c}_{jy}) \\ -\hat{c}_{jz} \end{bmatrix}$$

Refrax ratio is independent of rotation for Jacobian calculations

Experiments

Synthetic Data





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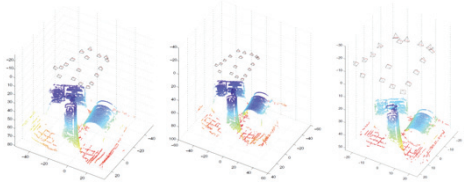
Real Data



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